LOW AND MEDIUM β SUPERCONDUCTING CAVITIES AND ACCELERATORS

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Introduction

- There have been increased needs for reduced-beta (β<1) SRF cavity especially in CW machine (or high duty pulsed machine; duty >10 %)

  - Accelerator driven system (ADS)
    Nuclear transmutation of long-lived radio active waste
    Energy amplifier
    Intense spallation neutron source

  - Nuclear physics
    Radioactive ion acceleration
    Muon/neutrino production

  - Military application
    Tritium production

- SRF technology → Critical path !!

Introduction

- Reduced beta Elliptical multi-cell SRF cavity
  - for CW, prototyping by several R&D groups demonstrate as low as β=0.47
  - for pulsed, SNS β=0.61, 0.81 cavities & ESS (SNS; cryomodule test progressing, production cavities are coming now)
- SRF cavity for CW application or long pulse application
  - efforts for expanding their application regions down to β~0.1,
- Elliptical cavity has intrinsic problem as β goes down
  - mechanical problem, multipacting, low RF efficiency
- Spoke cavity; supposed to cover ranges β=0.1~0.5(6), f=300~900 MHz
  - design & prototype efforts in RIA, AAA, EURISOL, XADS, ESS, etc.

For proton β=0.12 corresponds ~7 MeV → all the accelerating structures (except RFQ)

Low and Medium β Superconducting Accelerators

	High Current	Medium/Low Current
CW	Accelerator driven systems waste transmutation energy production	Production of radioactive ions Nuclear Structure
Pulsed	Pulsed spallation sources	

High-current cw accelerators

- Beam: p, H^-, d
- Technical issues and challenges
  - Beam losses (~ 1 W/m)
  - Activation
  - High cw rf power
  - Higher order modes
  - Cryogenics losses
- Implications for SRF technology
  - Cavities with high acceptance
  - Development of high cw power couplers
  - Extraction of HOM power
  - Cavities with high shunt impedance

High-current pulsed accelerators

- Beam: p, H^-
- Technical issues and challenges
  - Beam losses (~ 1 W/m)
  - Activation
  - Higher order modes
  - High peak rf power
  - Dynamic Lorentz detuning
- Implications for SRF technology
  - Cavities with high acceptance
  - Development of high peak power couplers
  - Extraction of HOM power
  - Development of active compensation of dynamic Lorentz detuning

Medium to low current cw accelerators

- Beam; p to U
- Technical issues and challenges
  - Microphonics, frequency control
  - Cryogenic losses
  - Wide charge to mass ratio
  - Multicharged state acceleration
  - Activation
- Implications for SRF technology
  - Cavities with low sensitivity to vibration
  - Development of microphonics compensation
  - Cavities with high shunt impedance
  - Cavities with large velocity acceptance (few cells)
  - Cavities with large beam acceptance (low frequency, small frequency transitions)

Common considerations (I)

- Intermediate velocity applications usually do not require (or cannot afford) very high gradients
- Operational and practical gradients are limited by
  - Cryogenics losses (cw applications)
  - Rf power to control microphonics (low current applications)
  - Rf power couplers (high-current applications)
- High shunt impedance is often more important
- To various degrees, beam losses and activation are a consideration

Common considerations (II)

- Superconducting accelerators in the medium velocity range are mostly used for the production of secondary species
  - Neutrons (spallation sources)
  - Exotic ions (radioactive beam facilities)
- Medium power (100s kW) to high power (~MW) primary impinging on a target
- Thermal properties and dynamics of the target are important considerations in the design of the accelerator (frequency, duration, recovery from beam trips)
- Some implications:
  - Operate cavities sufficiently far from the edge
  - Provide an ample frequency control window

Design considerations

- Low cryogenics losses
  - High $QR_s * R_{sh}/Q$
  - Low frequency
- High gradient
  - Low E_p/E_{acc}
  - Low B_p/E_{acc}
- Large velocity acceptance
  - Small number of cells
  - Low frequency
- Frequency control
  - Low sensitivity to microphonics
  - Low energy content
  - Low Lorentz coefficient
- Large beam acceptance
  - Large aperture (transverse acceptance)
  - Low frequency (longitudinal acceptance)

A Few Obvious Statements

Low and medium β

$\beta<1$

Particle velocity will change

The lower the velocity of the particle or cavity β

The faster the velocity of the particle will change

The narrower the velocity range of a particular cavity

The smaller the number of cavities of that β

The more important it is that the particle achieve design velocity

Be conservative at lower β

Be more aggressive at higher β

A Few More Statements

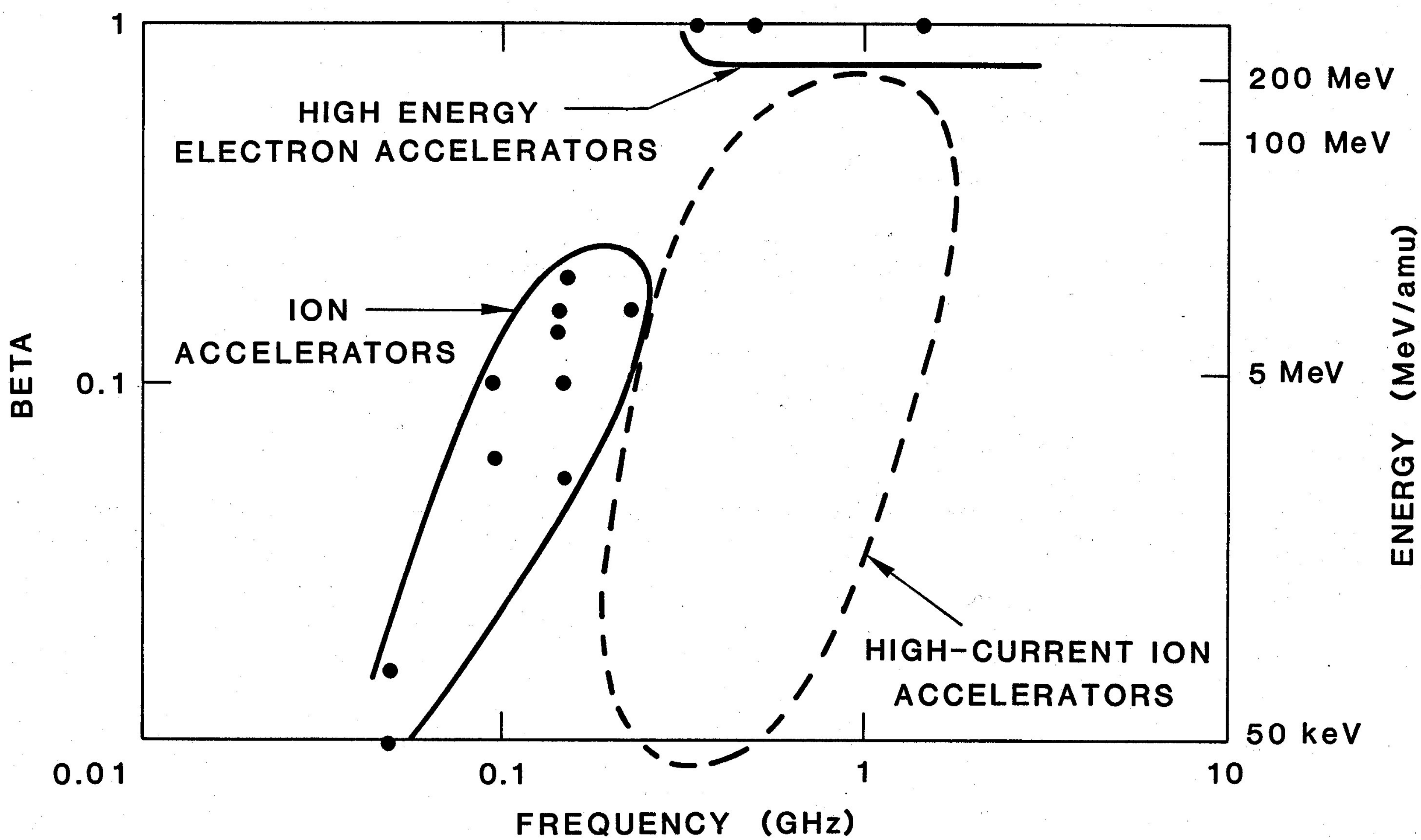
Two main types of structure geometries

- TEM class (QW, HW, Spoke)
- TM class (elliptical)

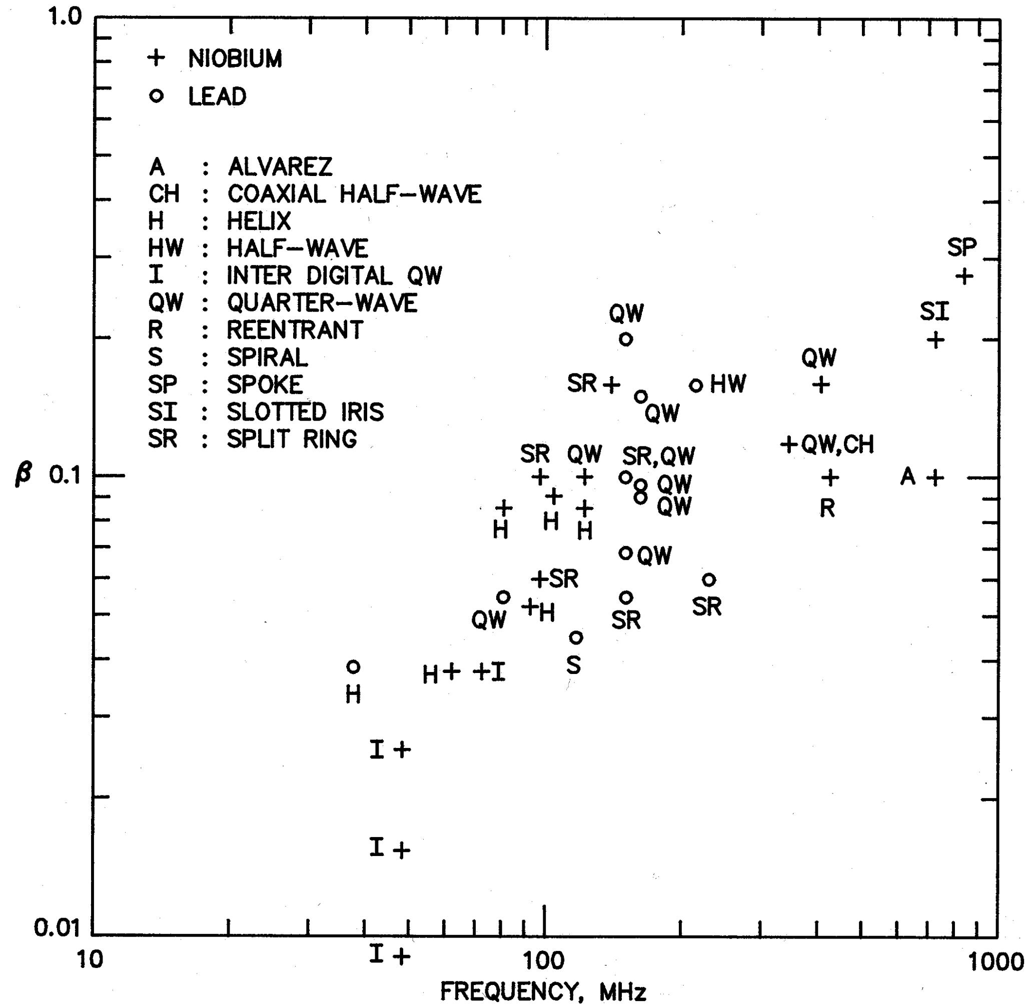
Design issues of medium β elliptical cavities are similar to those of β=1

- Most of the talk will be on TEM-class cavities
- For TM-class cavities see:
  - Design criteria for elliptical cavities
    Pagani, Barni, Bosotti, Pierini, Ciovati, SRF 2001.
  - Challenges and the future of reduced beta srf cavity design
    Sang-ho Kim, LINAC 2002.

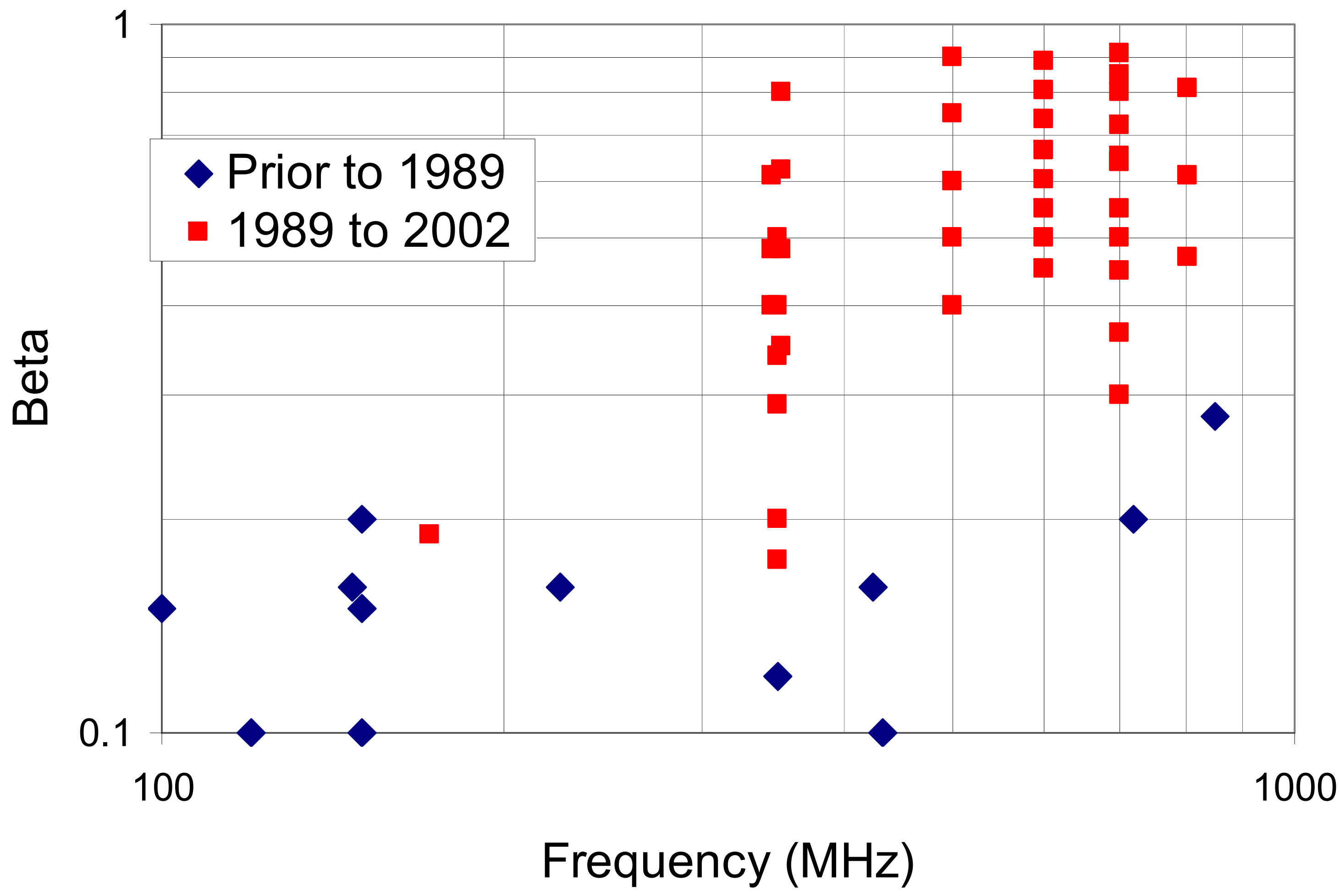
Superconducting Structures – Circa 1987



β<1 Superconducting Structures – Circa 1989



β<1 Superconducting Structures – 2002..



Basic Structure Geometries

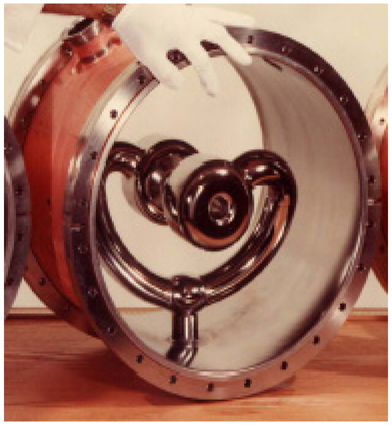
Resonant Transmission Lines

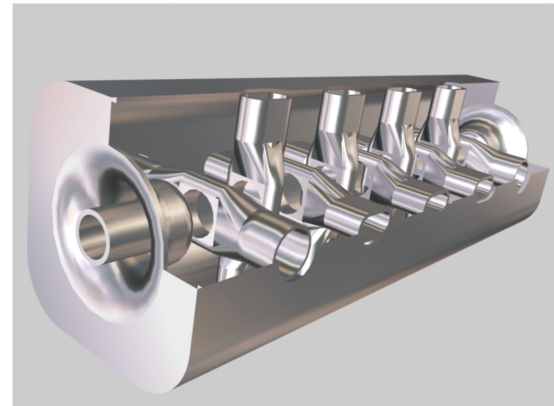
- $\lambda/4$
  - Quarter-wave
  - Split-ring
  - Twin quarter-wave
  - Lollipop
- $\lambda/2$
  - Coaxial half-wave
  - Spoke
  - H-types

- **TM**
  - Elliptical
  - Reentrant
- **Other**
  - Alvarez
  - Slotted-iris

A Word on Design Tools

TEM-class cavities are essentially 3D geometries





3D electromagnetic software is available

- MAFIA, Microwave Studio, HFSS, etc.

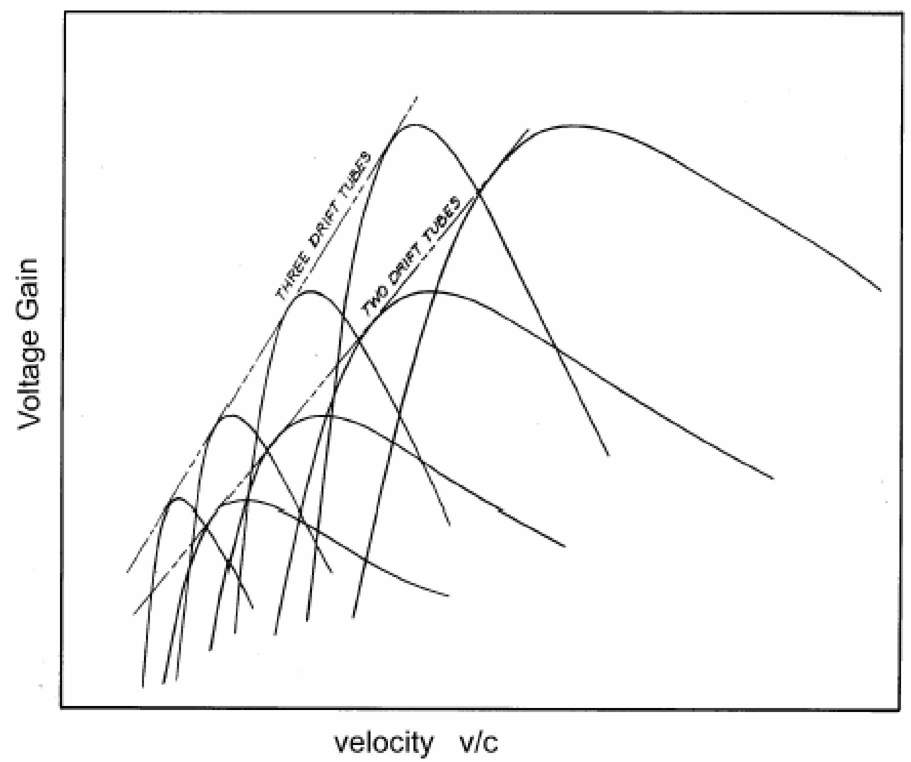
3D software is usually very good at calculating frequencies

- Not quite as good at calculating surface fields
  - Use caution, vary mesh size
  - Remember Electromagnetism 101

Design Tradeoffs

Number of cells

- Voltage gain
- Velocity acceptance



Frequency

- Size
- Voltage gain
- Rf losses
- Energy content, microphonics, rf control
- Acceptance, beam quality and losses

Energy Gain
Transit Time Factor - Velocity Acceptance

$$\Delta W = q \int_{-\infty}^{+\infty} E(z) \cos(\omega t + \phi)\, dz$$

- Assumption: constant velocity

$$\Delta W = q \ \cos\phi \ \Delta W_0 \ T(\beta) \qquad \Delta W_0 = \Theta \int_{-\infty}^{+\infty} |E(z)|\, dz$$

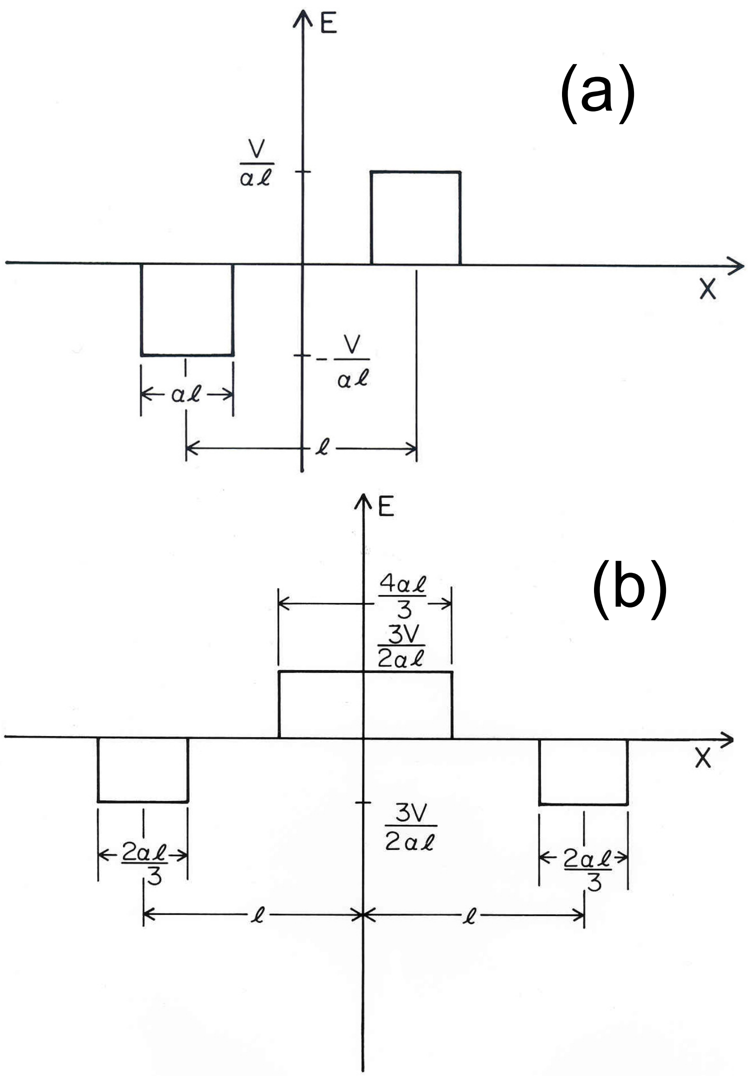
$$\Theta = \frac{\text{Max} \int_{-\infty}^{+\infty} E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz}{\int_{-\infty}^{+\infty} |E(z)|\, dz}$$

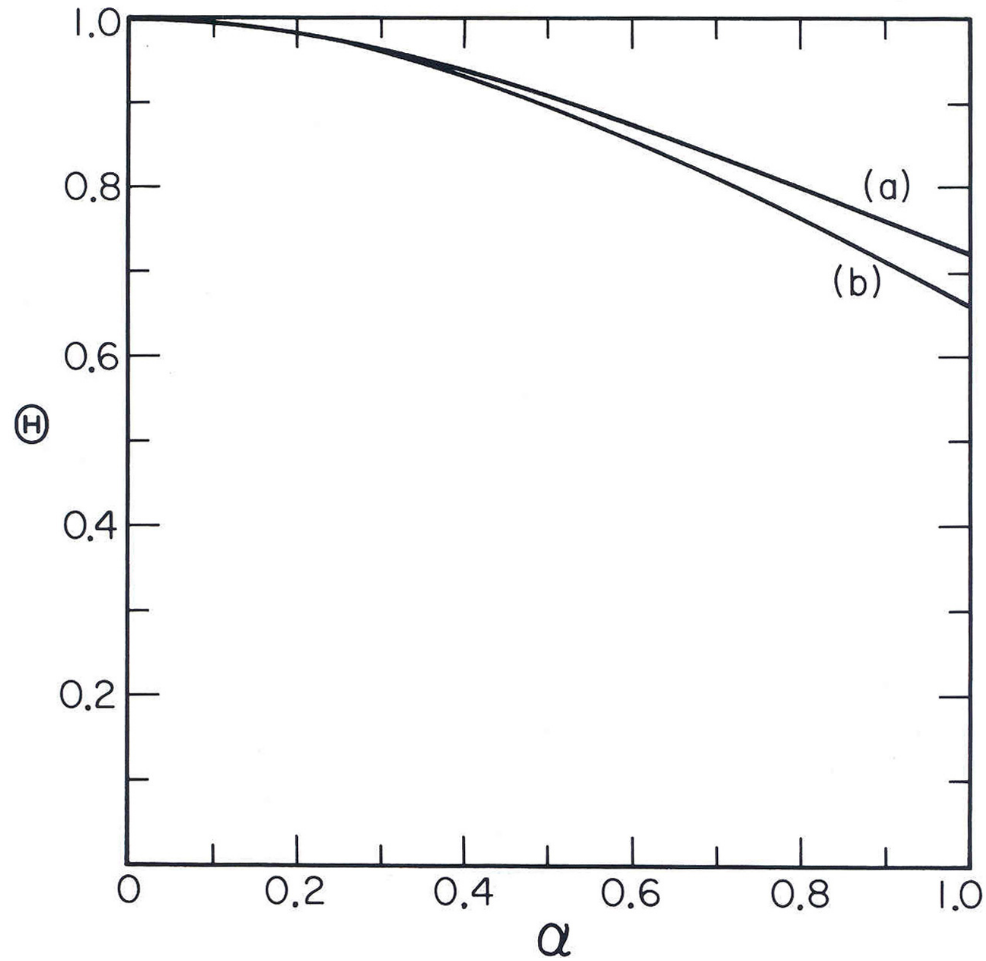
Transit Time Factor

$$T(\beta) = \frac{\int_{-\infty}^{+\infty} E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz}{\text{Max} \int_{-\infty}^{+\infty} E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz}$$

Velocity Acceptance

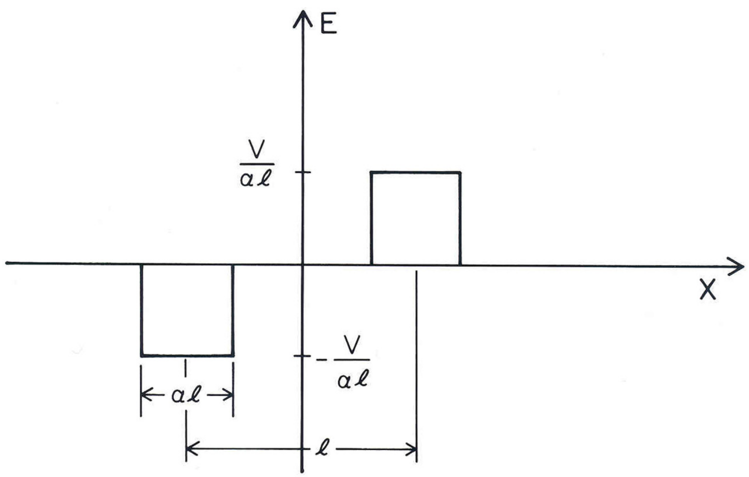
Transit Time Factor





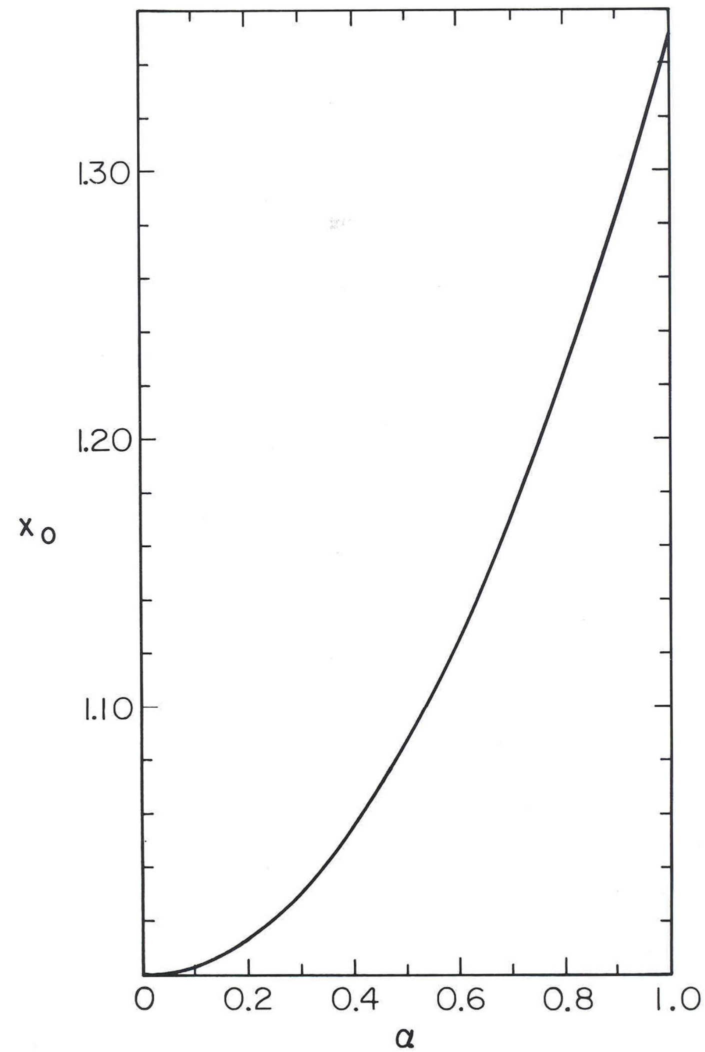
Velocity Acceptance for 2-Gap Structures

$$T(\beta)=\frac{\beta}{\beta_0}\frac{\sin\left(\frac{\pi\alpha}{2x_0}\frac{\beta_0}{\beta}\right)\sin\left(\frac{\pi}{2x_0}\frac{\beta_0}{\beta}\right)}{\sin\left(\frac{\pi\alpha}{2x_0}\right)\sin\left(\frac{\pi}{2x_0}\right)}$$



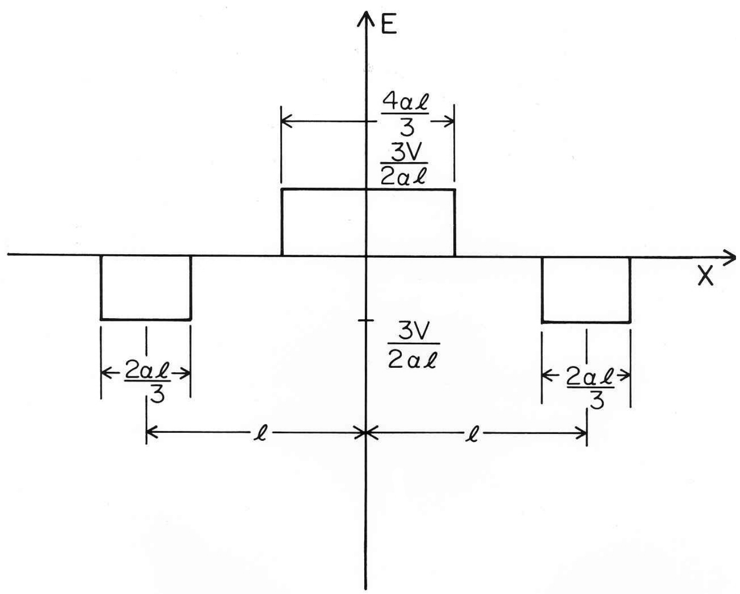
$$x_0=\frac{\beta_0\lambda}{2l}$$

$$l\simeq\frac{\beta_0\lambda}{2.2}$$

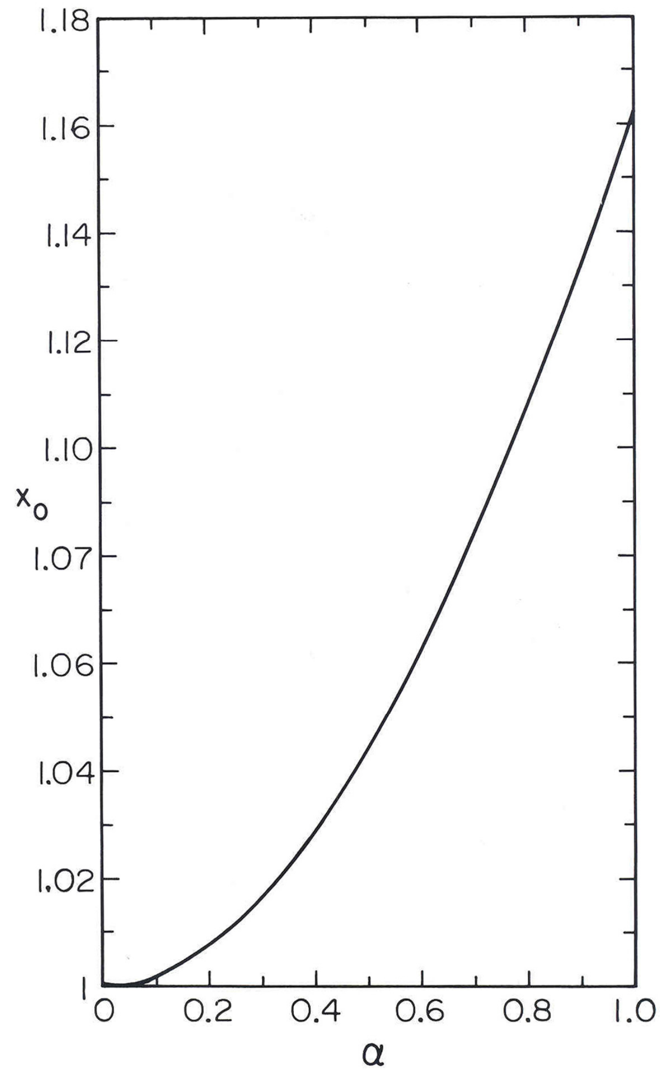


Velocity Acceptance for 3-Gap Structures

$$T(\beta)=\frac{\beta}{\beta_0}\frac{\sin\left(\frac{\pi\alpha}{3x_0}\frac{\beta_0}{\beta}\right)\left[\cos\left(\frac{\pi\alpha}{3x_0}\frac{\beta_0}{\beta}\right)-\cos\left(\frac{\pi}{x_0}\frac{\beta_0}{\beta}\right)\right]}{\sin\left(\frac{\pi\alpha}{3x_0}\right)\left[\cos\left(\frac{\pi\alpha}{3x_0}\right)-\cos\left(\frac{\pi}{x_0}\right)\right]}$$



$$x_0=\frac{\beta_0\lambda}{2l}$$

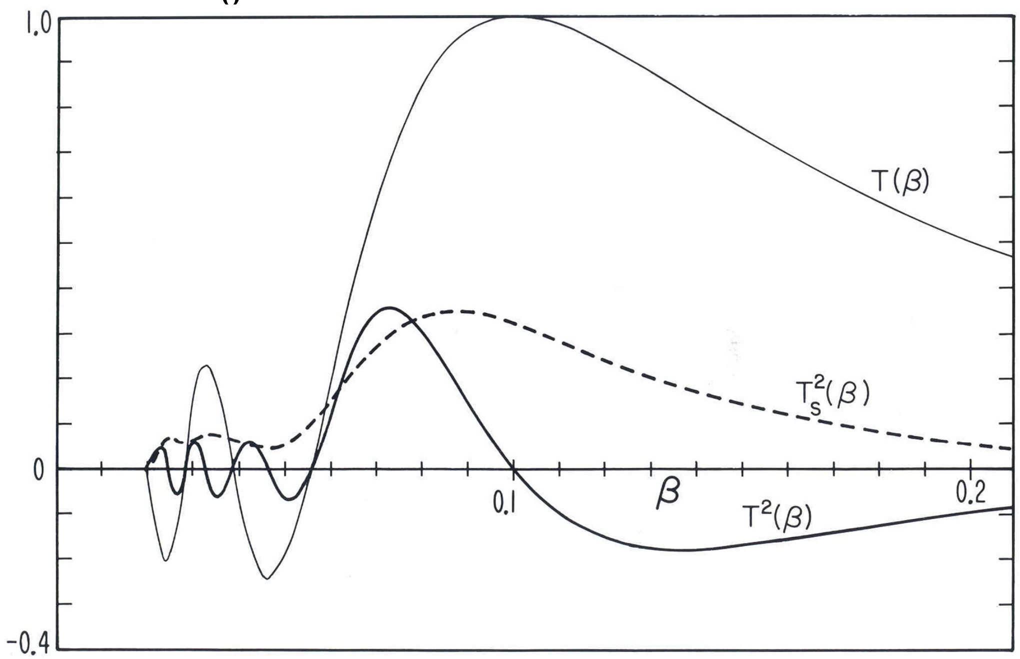


Higher-Order Effects

$$\Delta W = q\cos\phi\,\Delta W_0\,T(\beta) + \frac{(q\Delta W_0)^2}{W}\left[T^{(2)}(\beta) + \sin 2\phi\,T_s^{(2)}(\beta)\right]$$

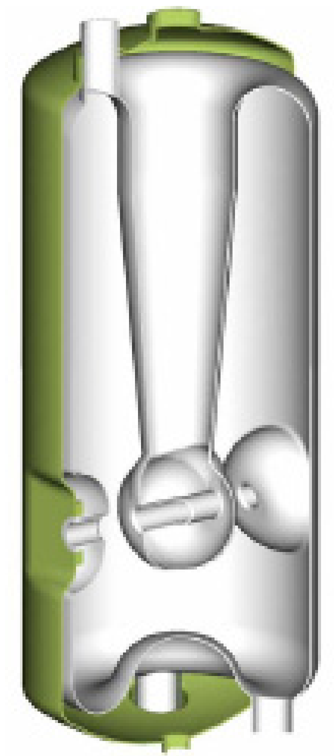
$$T^{(2)}(k) = -\frac{k}{4}T(k)\frac{d}{dk}T(k) \qquad\qquad k = \omega/\beta c$$

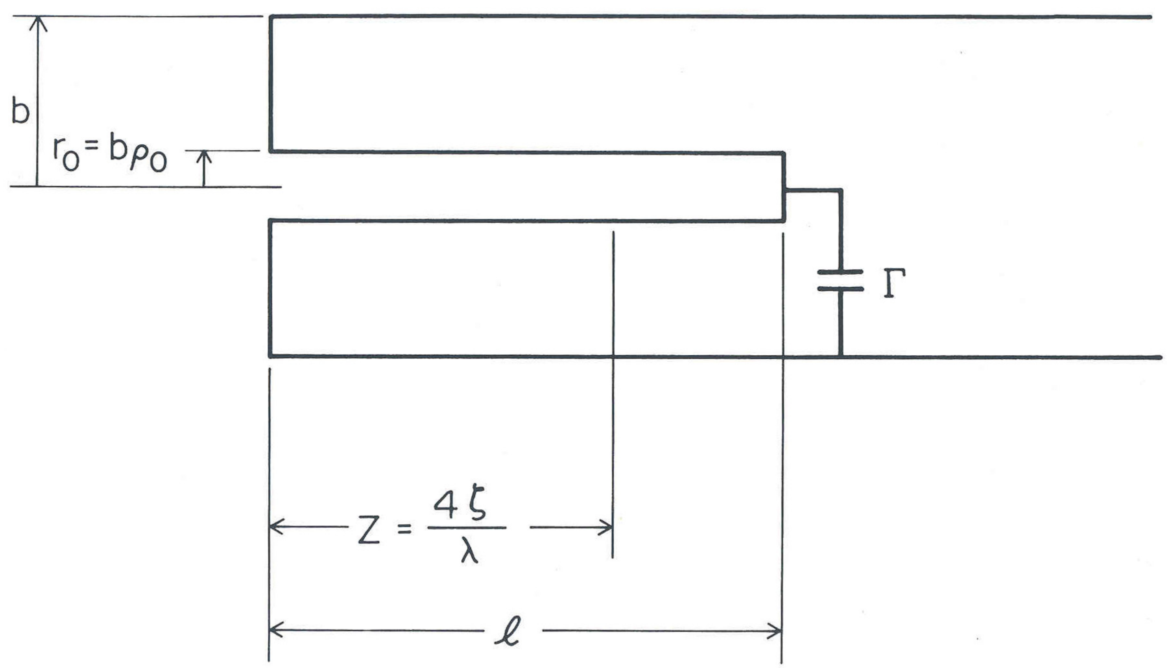
$$T_s^{(2)}(k) = -\frac{k}{4\pi}\int_0^\infty \frac{T(k+k')T(k-k') - T(k)T(k)}{k'^2}dk'$$



A Simple Model:
Loaded Quarter-wavelength Resonant Line

If characteristic length <<λ (β<0.5), separate the problem in two parts:
- Electrostatic model of high voltage region
- Transmission line

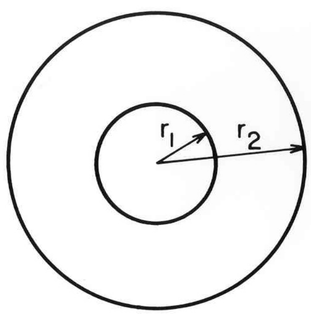


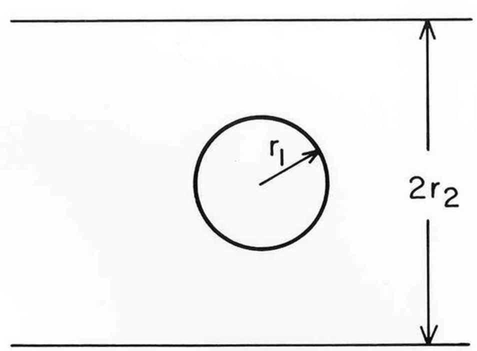


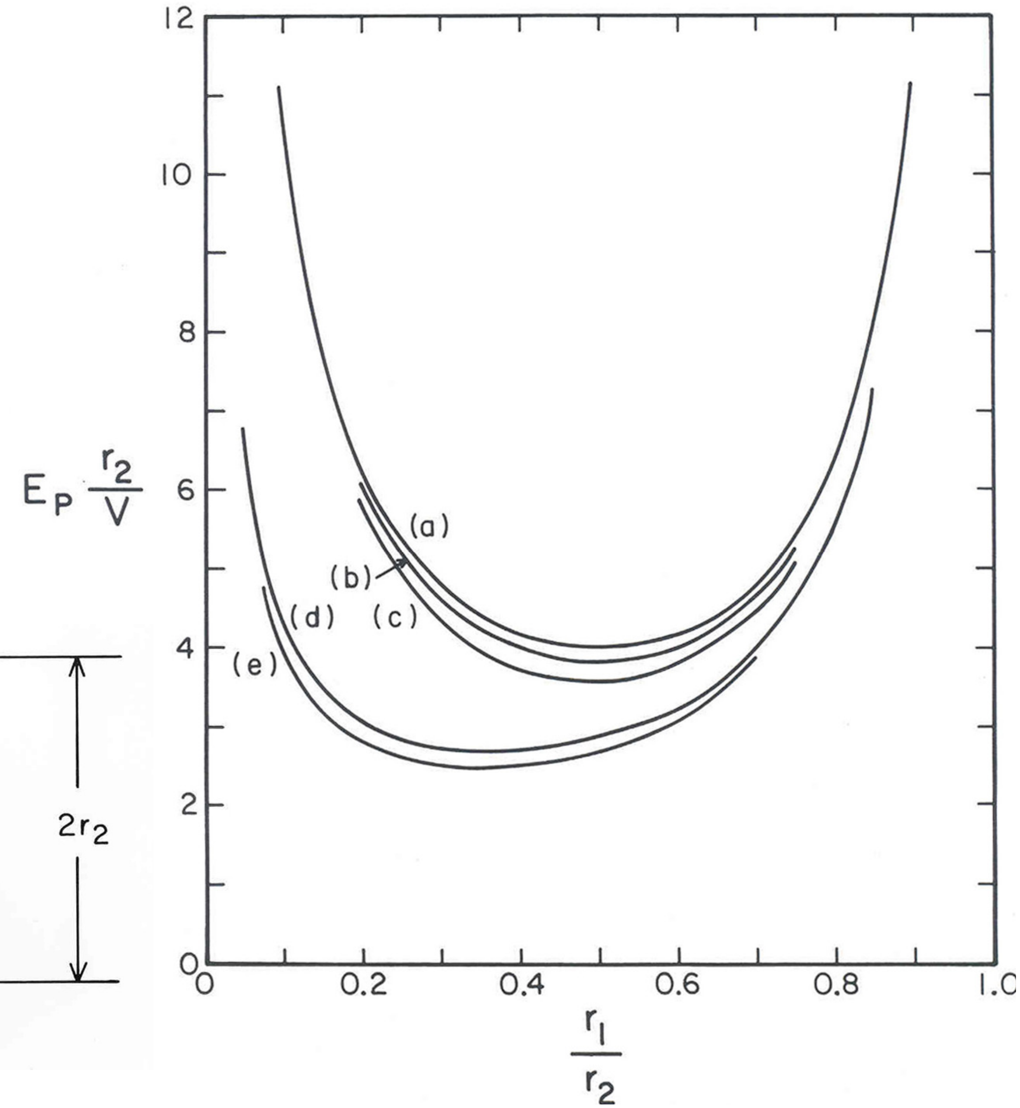
Basic Electrostatics

a: concentric spheres
b: sphere in cylinder
c: sphere between 2 planes
d: coaxial cylinders
e: cylinder between 2 planes

V_p : Voltage on center conductor
Outer conductor at ground
E_p: Peak field on center conductor



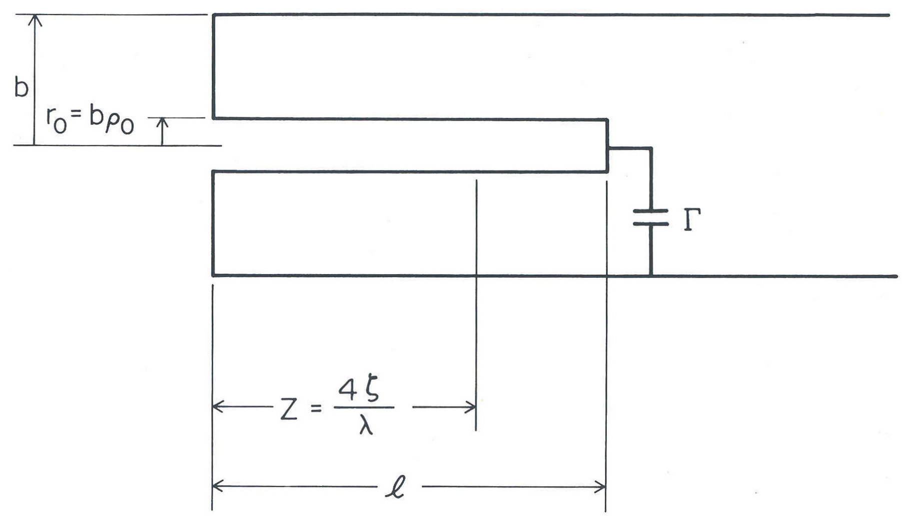




Loaded Quarter-wavelength Resonant Line

Capacitance per unit length

$$C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{r_0}\right)} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{1}{\rho_0}\right)}$$



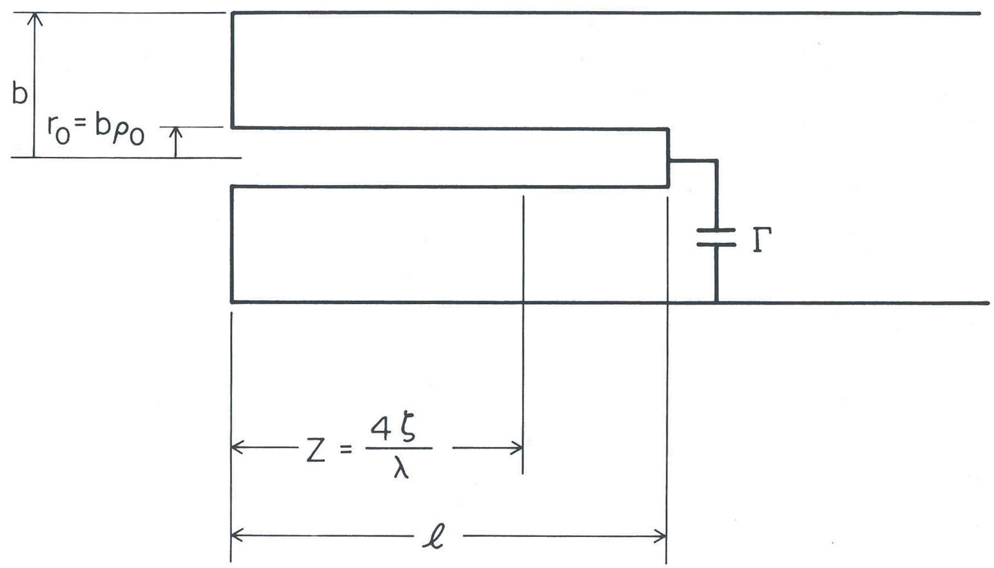
Inductance per unit length

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{r_0}\right) = \frac{\mu_0}{2\pi}\ln\left(\frac{1}{\rho_0}\right)$$

Loaded Quarter-wavelength Resonant Line

Center conductor voltage

$$V(z) = V_0 \sin\left(\frac{2\pi}{\lambda} z\right)$$



Center conductor current

$$I(z) = I_0 \cos\left(\frac{2\pi}{\lambda} z\right)$$

Line impedance

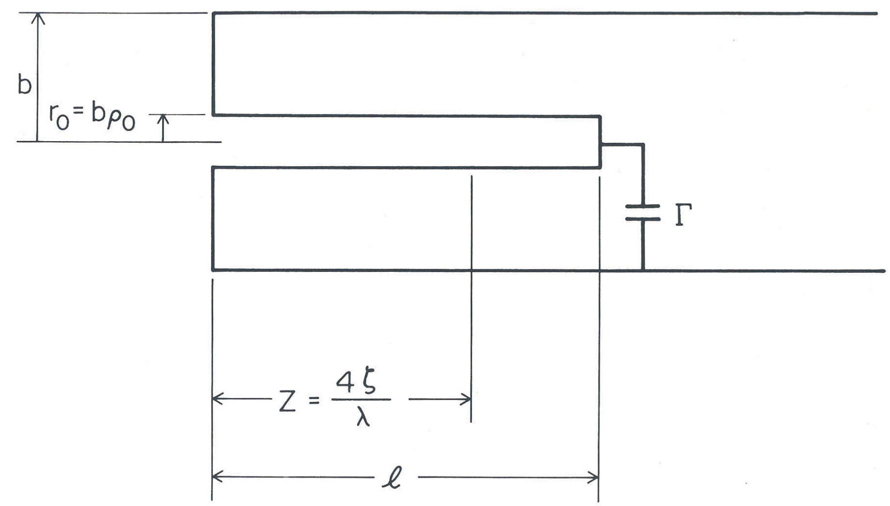
$$Z_0 = \frac{V_0}{I_0} = \frac{\eta}{2\pi} \ln\left(\frac{1}{\rho_0}\right), \qquad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 377\Omega$$

Loaded Quarter-wavelength Resonant Line

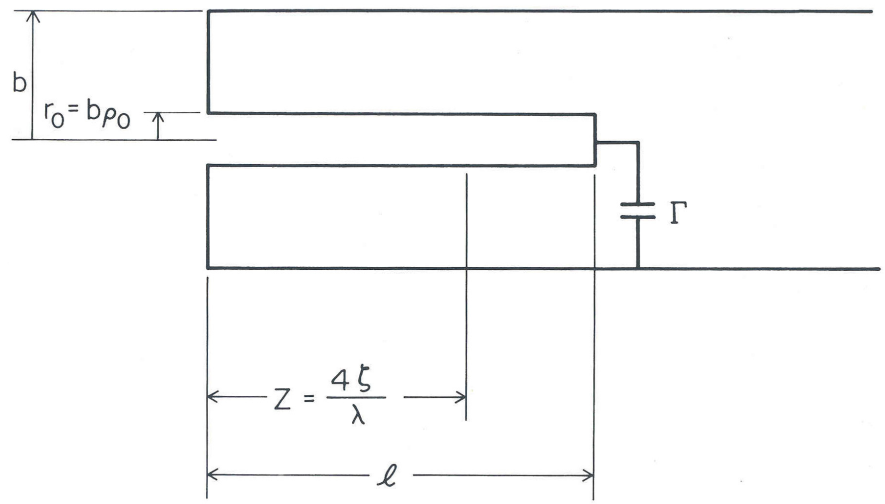
Loading capacitance

$$\Gamma(z) = \lambda\varepsilon \frac{\text{cotan}\left(\frac{2\pi}{\lambda} z\right)}{\ln\left(1/r_0\right)} = \lambda\varepsilon \frac{\text{cotan}\left(\frac{\pi}{2}\zeta\right)}{\ln\left(1/\rho_0\right)}$$

$$l = \frac{\lambda}{2\pi} \text{Arctan}\left[\frac{\lambda\varepsilon}{\Gamma \ln\left(1/\rho_0\right)}\right]$$



Loaded Quarter-wavelength Resonant Line



Peak magnetic field

$$\frac{V_p}{b} = \begin{Bmatrix} \eta & H \\ c & B \\ 300 & B \end{Bmatrix} \rho_0 \ln\left(\frac{1}{\rho_0}\right) \sin\left(\frac{\pi}{2}\zeta\right) \qquad \begin{Bmatrix} \text{m, A/m} \\ \text{m, T} \\ \text{cm, G} \end{Bmatrix}$$

V_p: Voltage across loading capacitance

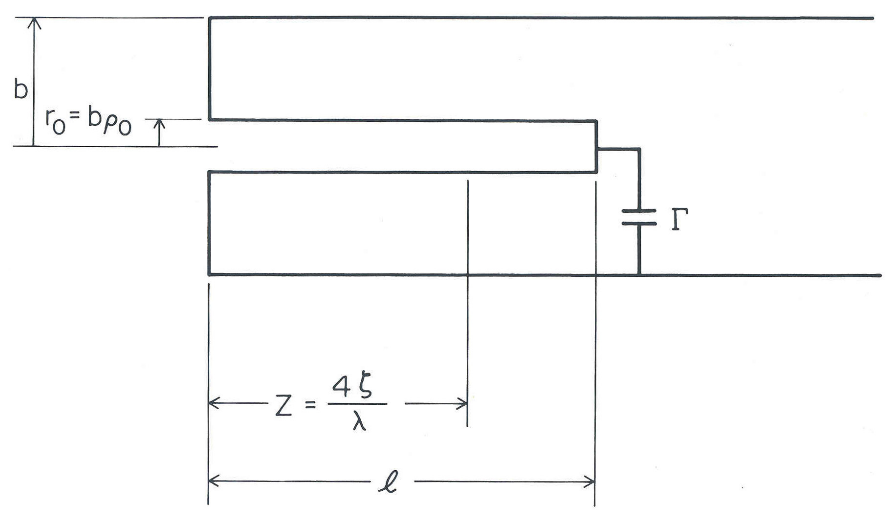
$B \simeq 9$ mT at 1 MV/m

Loaded Quarter-wavelength Resonant Line

Power dissipation (ignore losses in the shorting plate)

$$P = V_p^2 \frac{8}{\pi} \frac{R_s}{\eta^2} \frac{\lambda}{b} \frac{1+1/\rho_0}{\ln^2 \rho_0} \frac{\zeta + \frac{1}{\pi}\sin \pi\zeta}{\sin^2 \frac{\pi}{2}\zeta}$$

$$P \propto \frac{R_s}{\eta^2} E^2 \beta \lambda^2$$

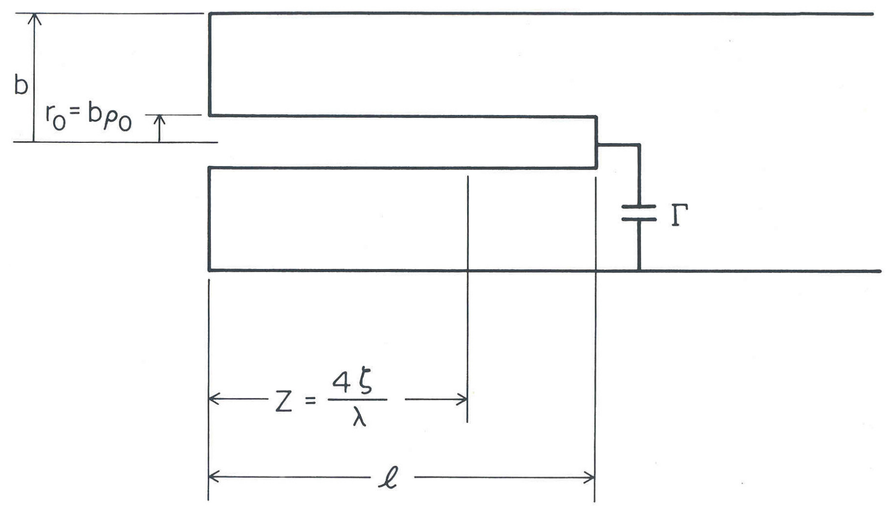


Loaded Quarter-wavelength Resonant Line

Energy content

$$U = V_p^2 \frac{\pi\varepsilon_0}{8} \lambda \frac{1}{\ln(1/\rho_0)} \frac{\zeta + \frac{1}{\pi}\sin\pi\zeta}{\sin^2 \frac{\pi}{2}\zeta}$$

$$U \propto \varepsilon_0 E^2 \beta^2 \lambda^3$$

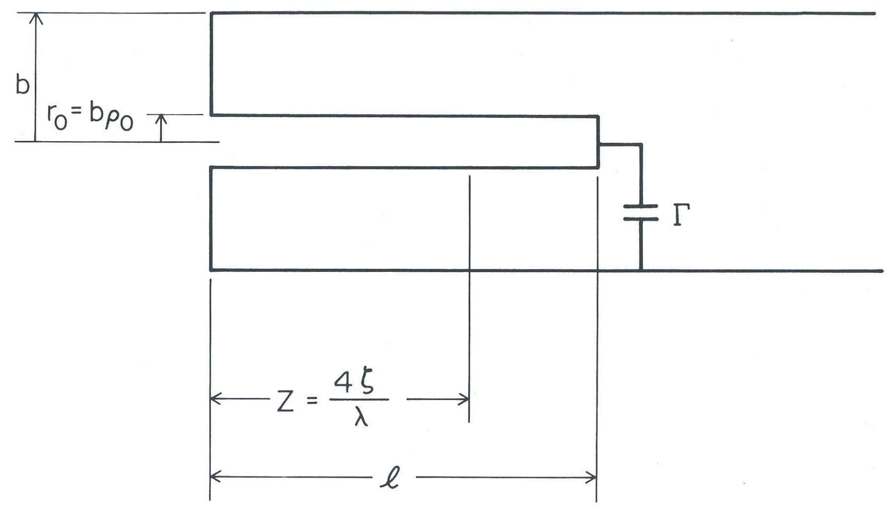


Loaded Quarter-wavelength Resonant Line

Geometrical factor

$$G = QR_s = 2\pi\,\eta\,\frac{b}{\lambda}\,\frac{\ln(1/\rho_0)}{1+1/\rho_0}$$

$$G \propto \eta\,\beta$$

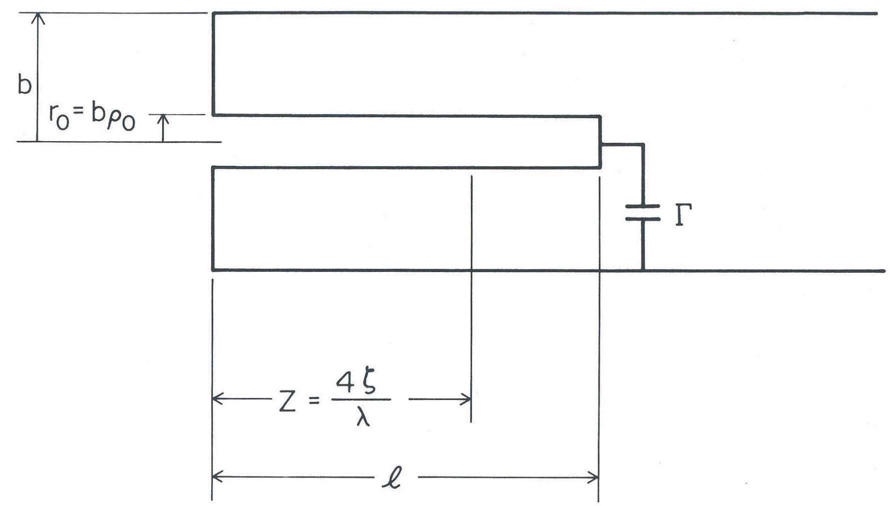


Loaded Quarter-wavelength Resonant Line

Shunt impedance $\left(4V_p^2 / P\right)$

$$R_{sh} = \frac{\eta^2}{R_s} \frac{32}{\pi} \frac{b}{\lambda} \frac{\ln^2 \rho_0}{1+1/\rho_0} \frac{\sin^2 \frac{\pi}{2}\zeta}{\zeta + \frac{1}{\pi}\sin \pi\zeta}$$

$$R_{sh}\, R_s \propto \eta^2\, \beta$$

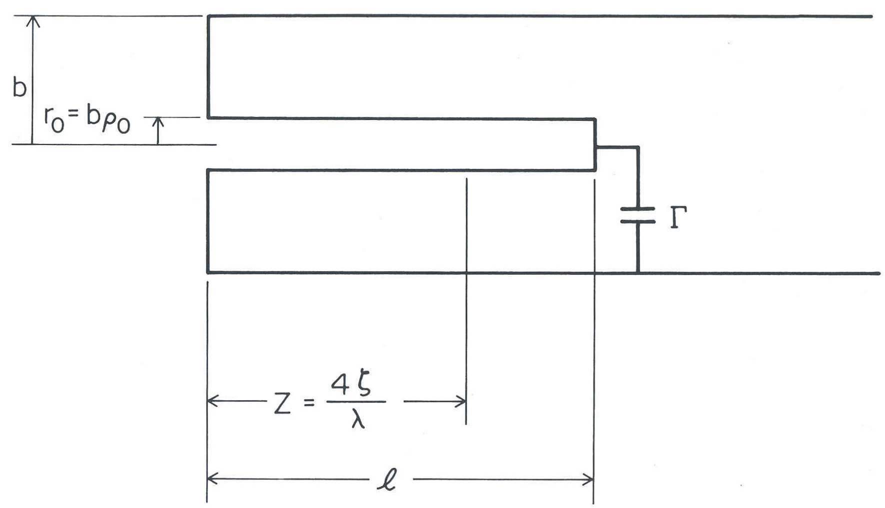


Loaded Quarter-wavelength Resonant Line

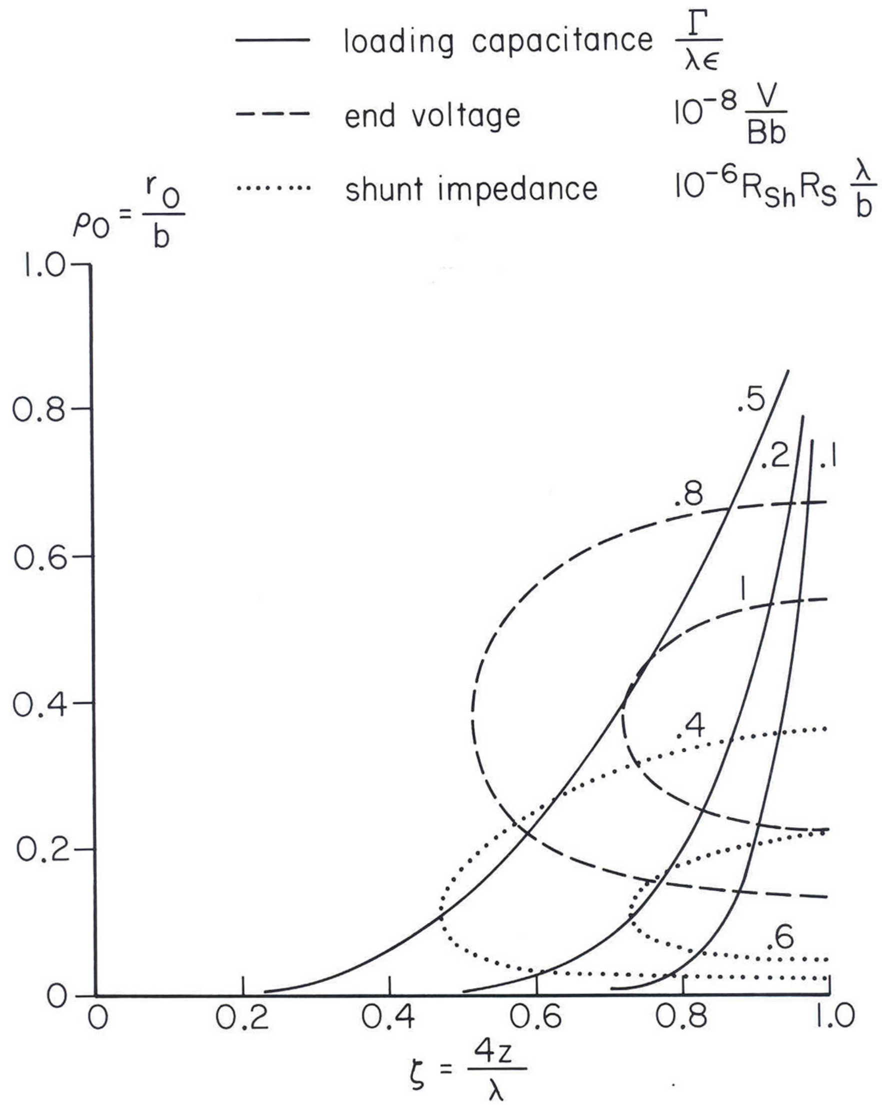
R/Q

$$\frac{R_{sh}}{Q} = \frac{16}{\pi^2}\,\eta \ln\left(1/\rho_0\right) \frac{\sin^2 \frac{\pi}{2}\zeta}{\zeta + \frac{1}{\pi}\sin \pi\zeta}$$

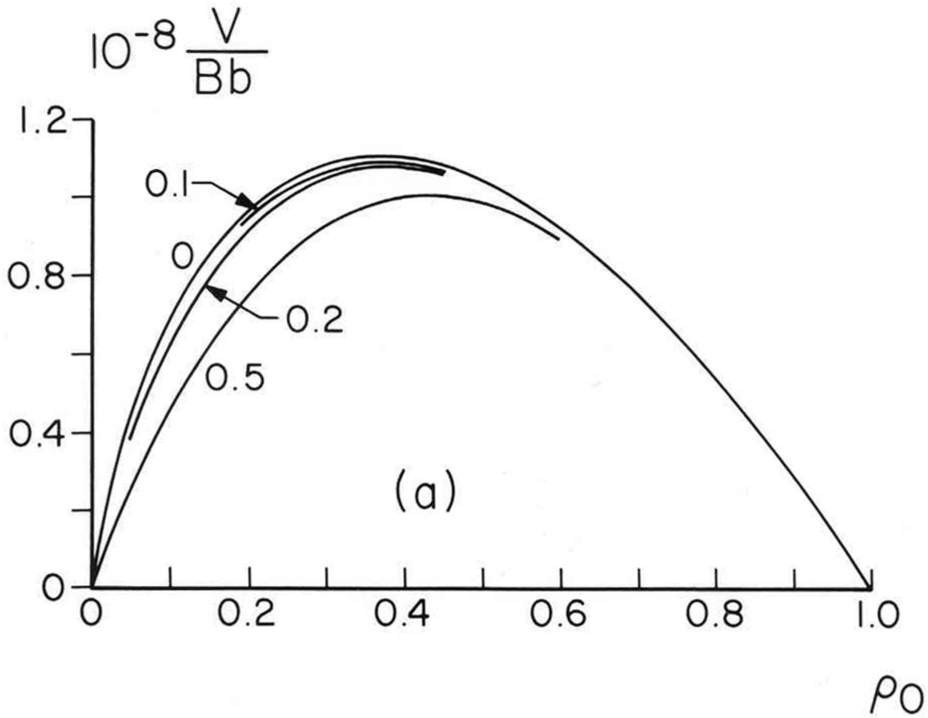
$$\frac{R_{sh}}{Q} \propto \eta$$

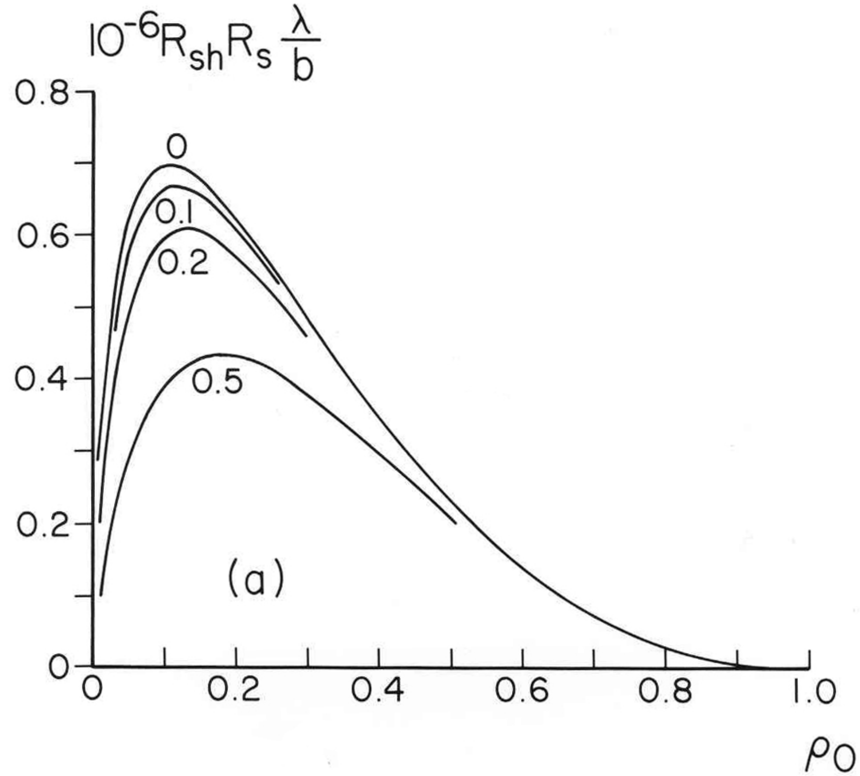


Loaded Quarter-wavelength Resonant Line



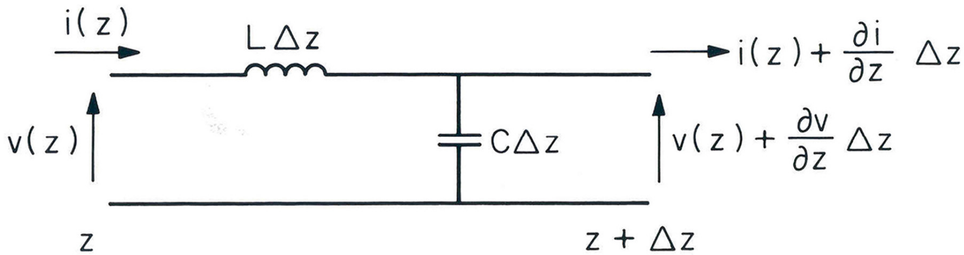
Loaded Quarter-wavelength Resonant Line





MKS units, lines of constant normalized loading capacitance $\Gamma/\lambda\varepsilon_0$

More Complicated Center Conductor Geometries



$$\frac{d^2v}{d\zeta^2} - \frac{1}{\rho \ln \rho}\frac{d\rho}{d\zeta}\frac{dv}{d\zeta} + \frac{\pi^2}{4}v = 0$$

$$\frac{d^2i}{d\zeta^2} + \frac{1}{\rho \ln \rho}\frac{d\rho}{d\zeta}\frac{di}{d\zeta} + \frac{\pi^2}{4}i = 0$$

$$\Gamma(z) = -C(z)\frac{i(z)}{di/dz}$$

More Complicated Center Conductor Geometries

Constant logarithmic derivative of line capacitance

Good model for linear taper

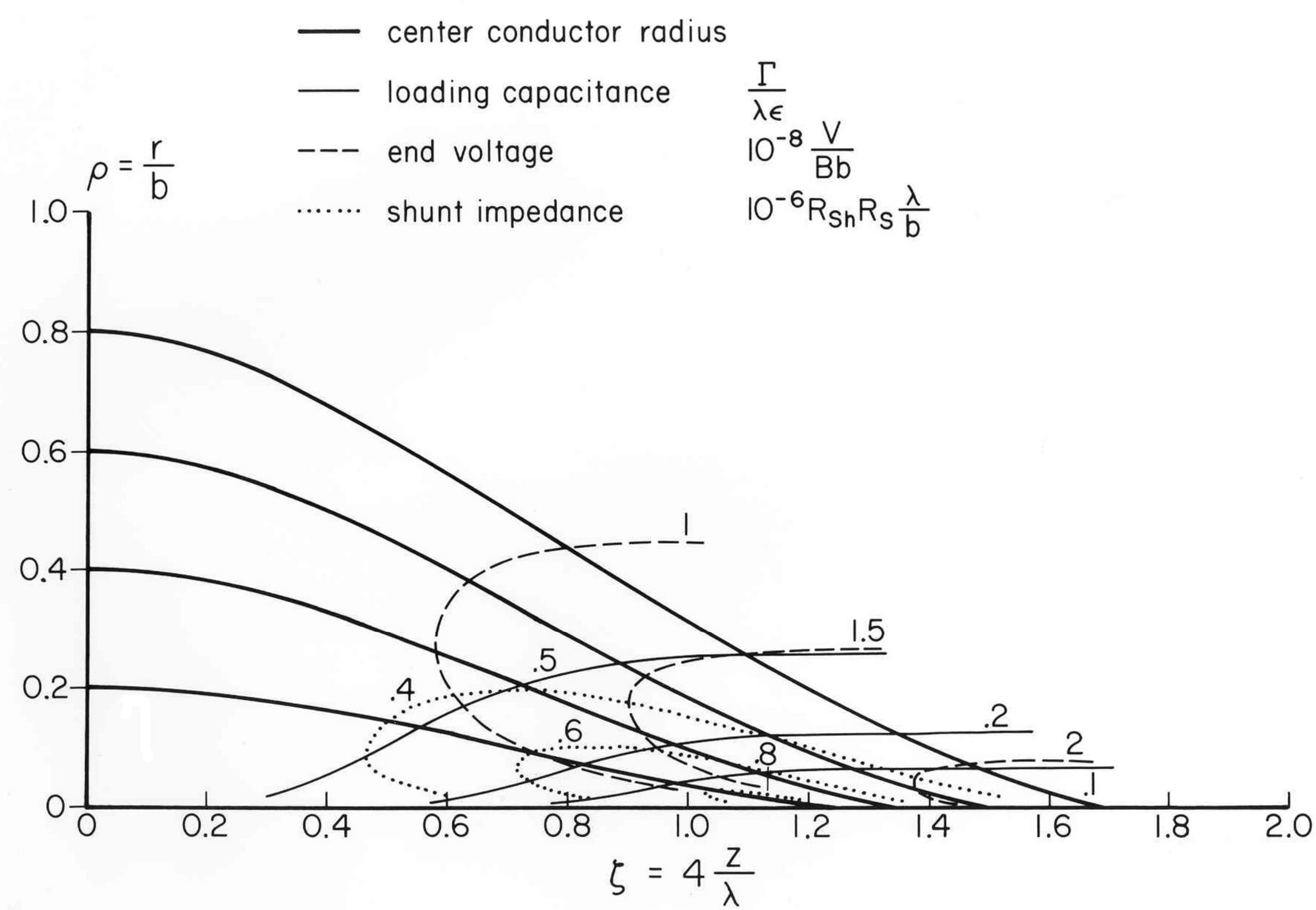
$$\frac{1}{C}\frac{dC}{dz} = -\frac{1}{d} \qquad\qquad r(z) = b\left(\frac{r_0}{b}\right)^{\exp(z/d)}$$

Constant surface magnetic field

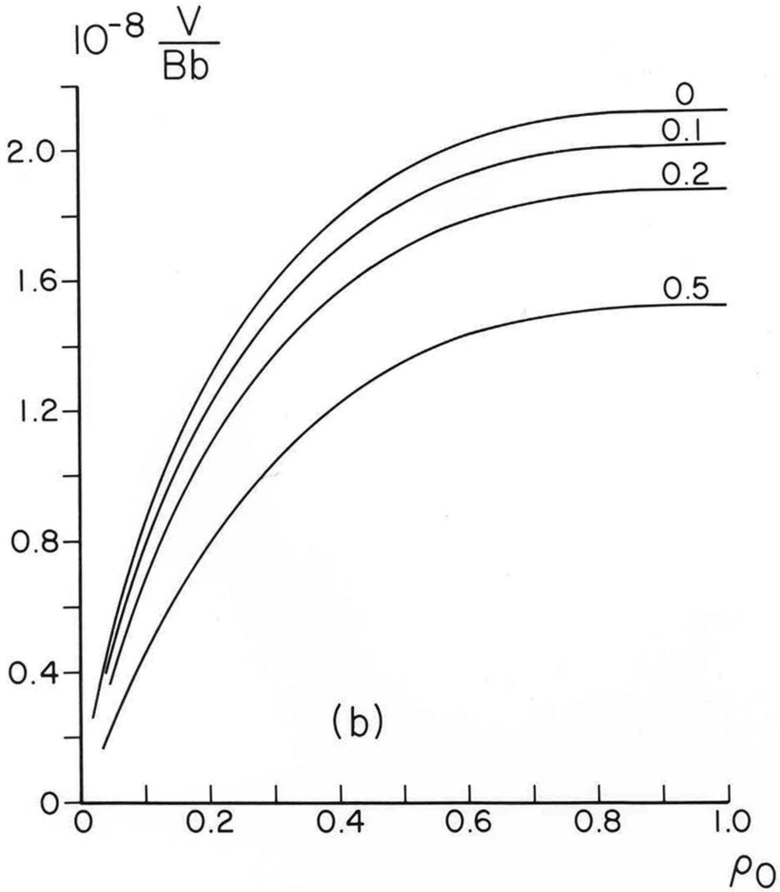
$$i(z) \propto r(z)$$

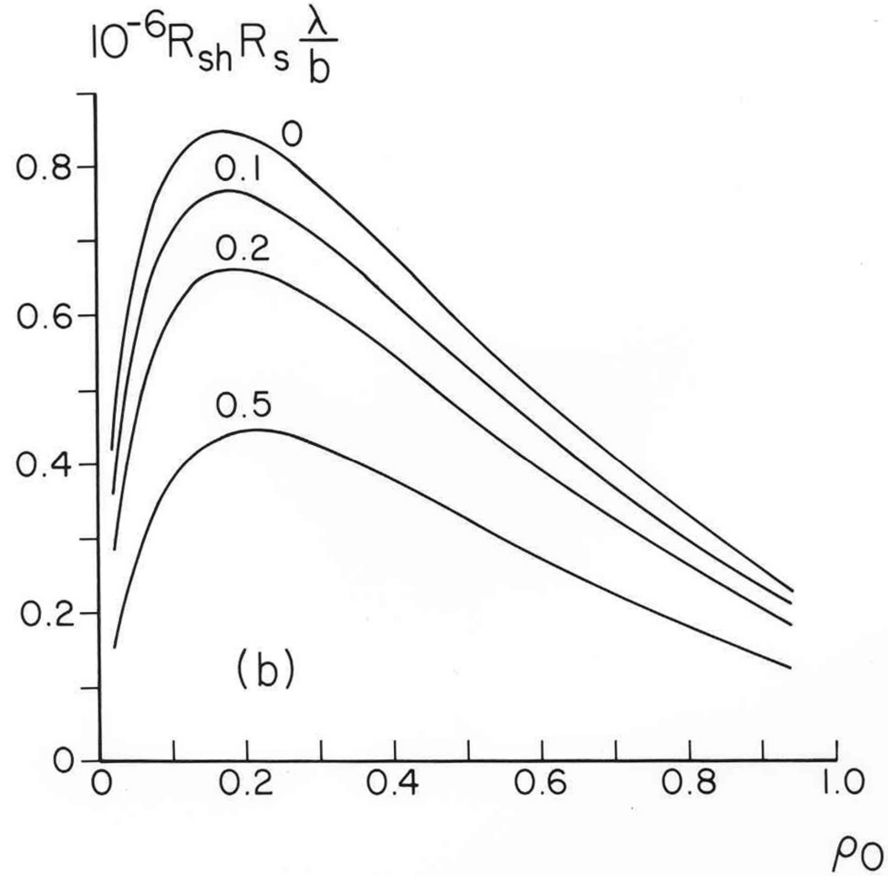
$$\frac{d^2 r}{dz^2} - \frac{1}{r\ln(b/r)}\left(\frac{dr}{dz}\right)^2 + \frac{4\pi^2}{\lambda^2} r = 0$$

Profile of Constant Surface Magnetic Field



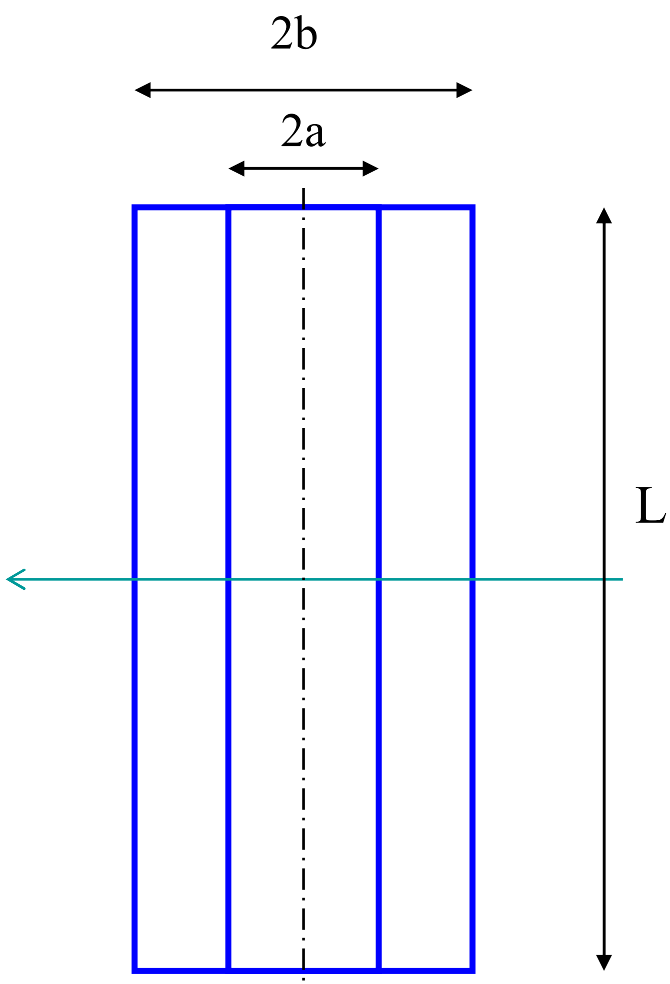
Profile of Constant Surface Magnetic Field

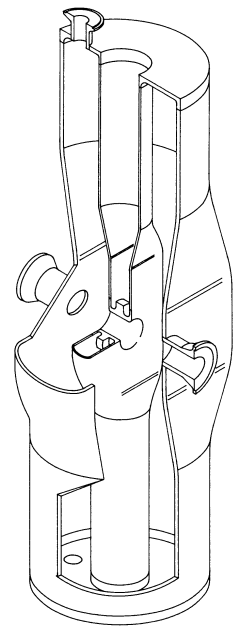


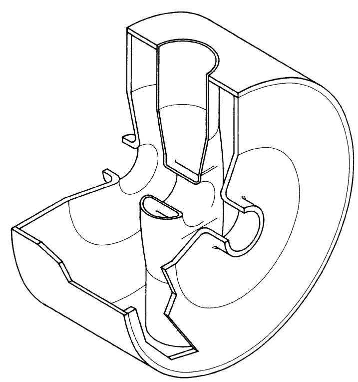


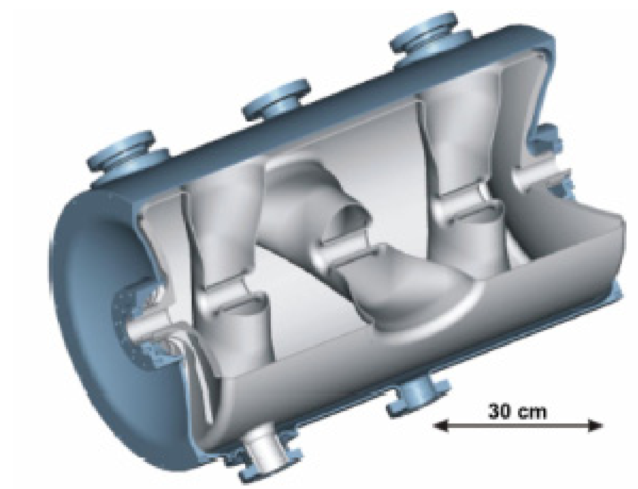
MKS units, lines of constant normalized loading capacitance $\Gamma/\lambda\varepsilon_0$

Another Simple Model:
Coaxial Half-wave Resonator









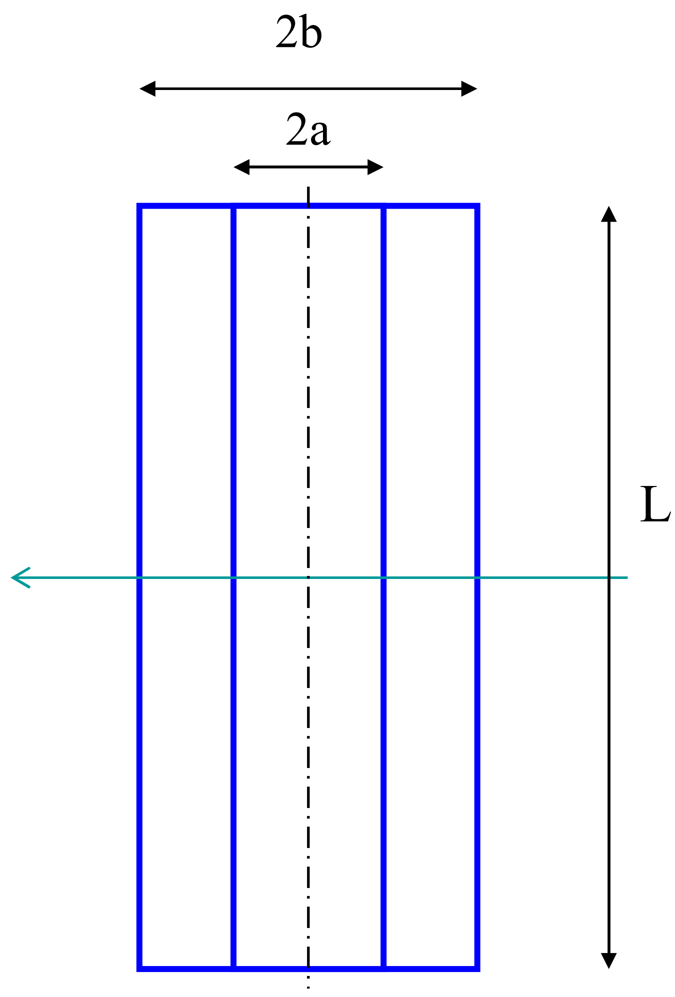
Coaxial Half-wave Resonator

Capacitance per unit length

$$C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{1}{\rho_0}\right)}$$

Inductance per unit length

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{r_0}\right) = \frac{\mu_0}{2\pi}\ln\left(\frac{1}{\rho_0}\right)$$



Coaxial Half-wave Resonator

Center conductor voltage

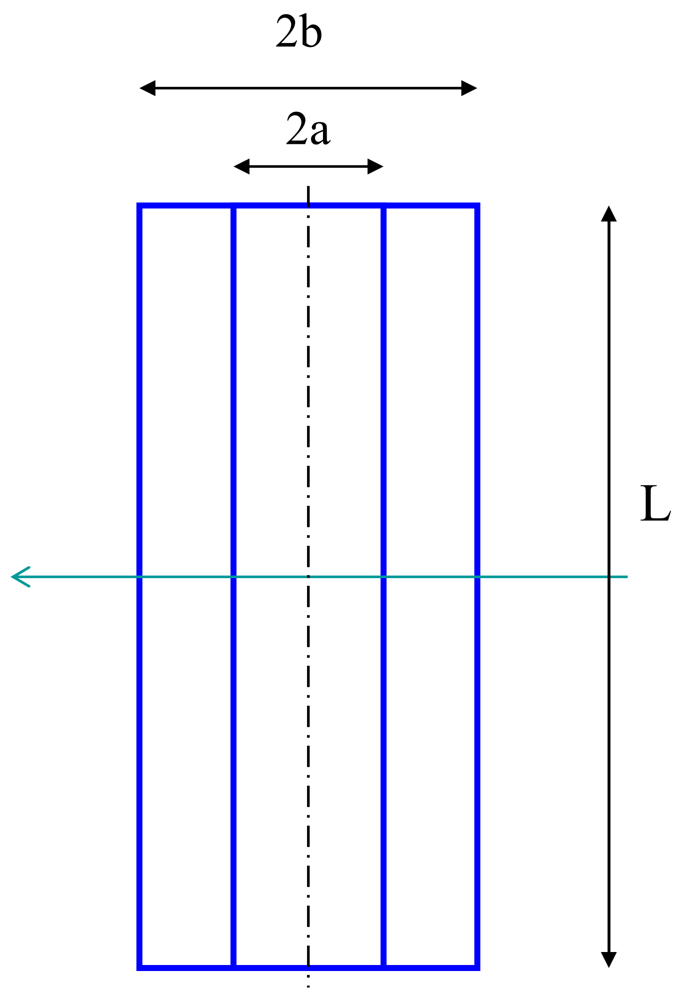
$$V(z) = V_0 \cos\left(\frac{2\pi}{\lambda} z\right)$$

Center conductor current

$$I(z) = I_0 \sin\left(\frac{2\pi}{\lambda} z\right)$$

Line impedance

$$Z_0 = \frac{V_0}{I_0} = \frac{\eta}{2\pi} \ln\left(\frac{1}{\rho_0}\right) , \qquad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 377\Omega$$

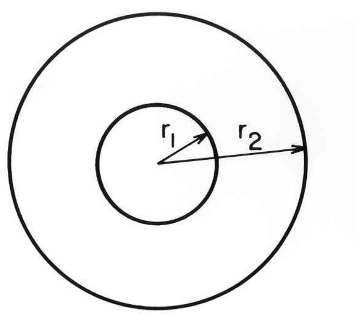


Coaxial Half-wave Resonator

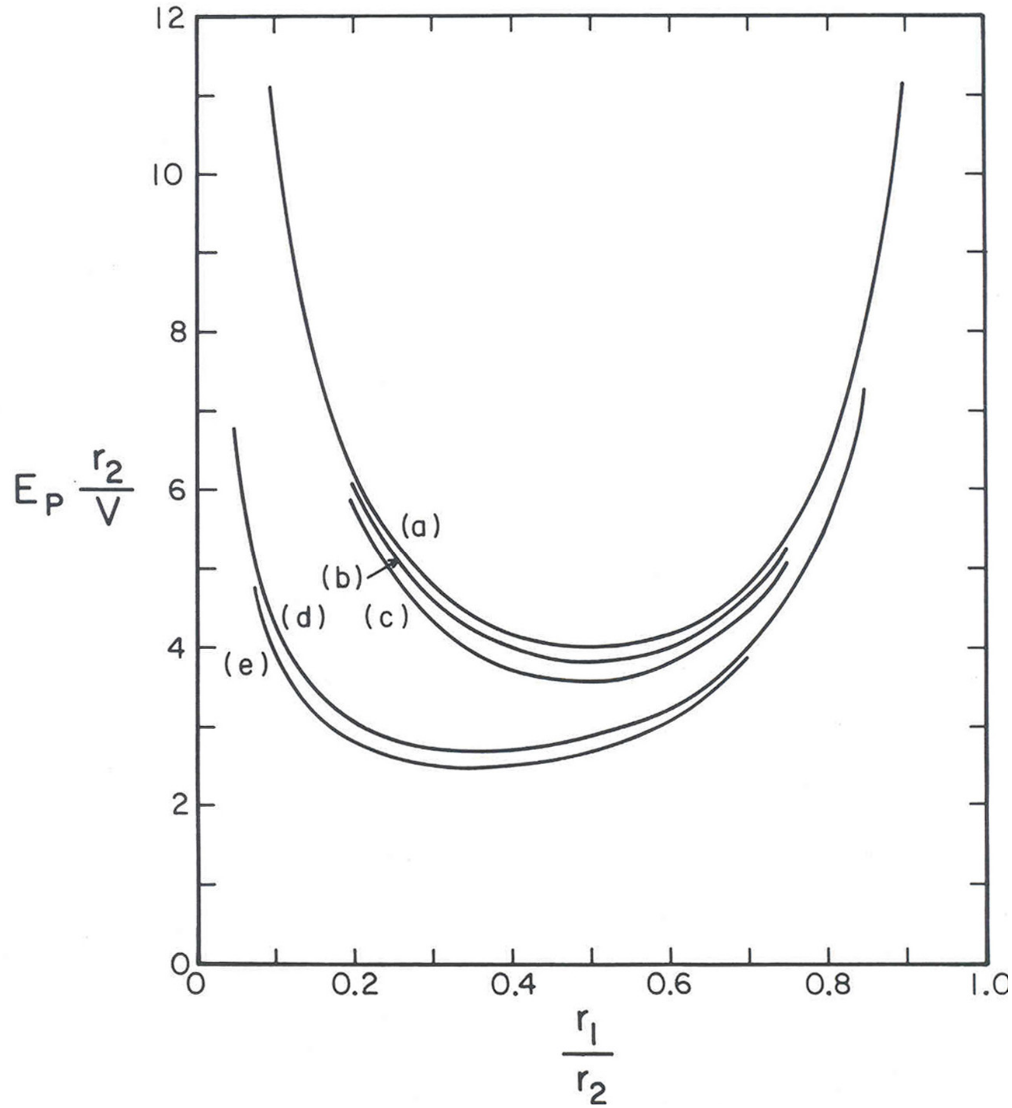
Peak Electric Field

d: coaxial cylinders

V_p : Voltage on center conductor
Outer conductor at ground
E_p: Peak field on center conductor



(a), (b), (d)



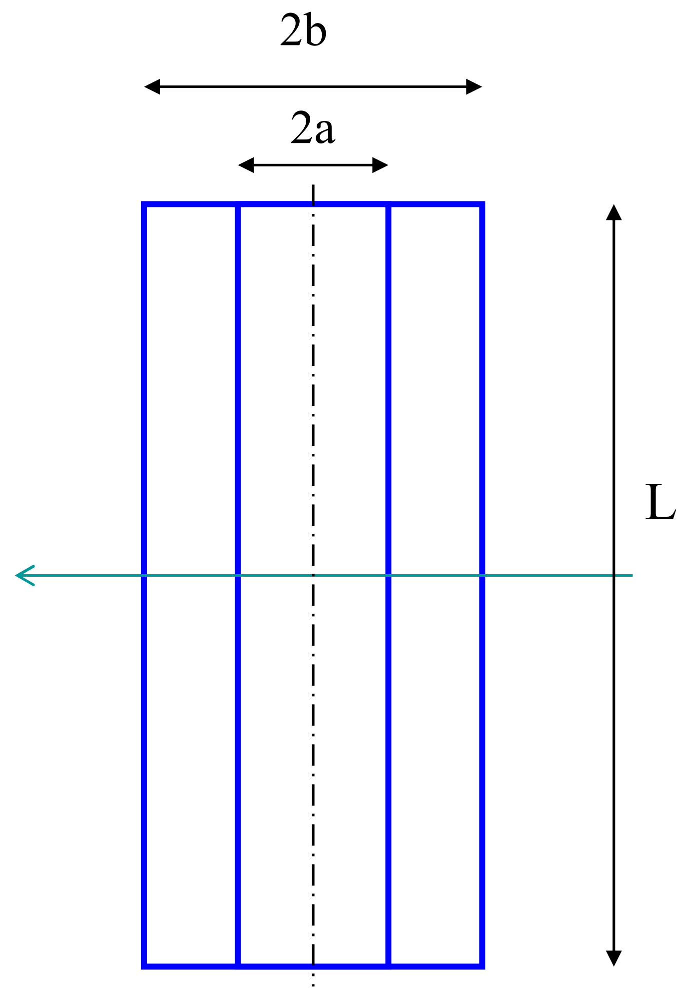
Coaxial Half-wave Resonator

Peak magnetic field

$$\frac{V_p}{b} = \begin{Bmatrix} \eta & H \\ c & B \\ 300 & B \end{Bmatrix} \rho_0 \ln\left(\frac{1}{\rho_0}\right) \qquad \begin{Bmatrix} \text{m, A/m} \\ \text{m, T} \\ \text{cm, G} \end{Bmatrix}$$

V_p: Voltage across loading capacitance

$B \simeq 9$ mT at 1 MV/m

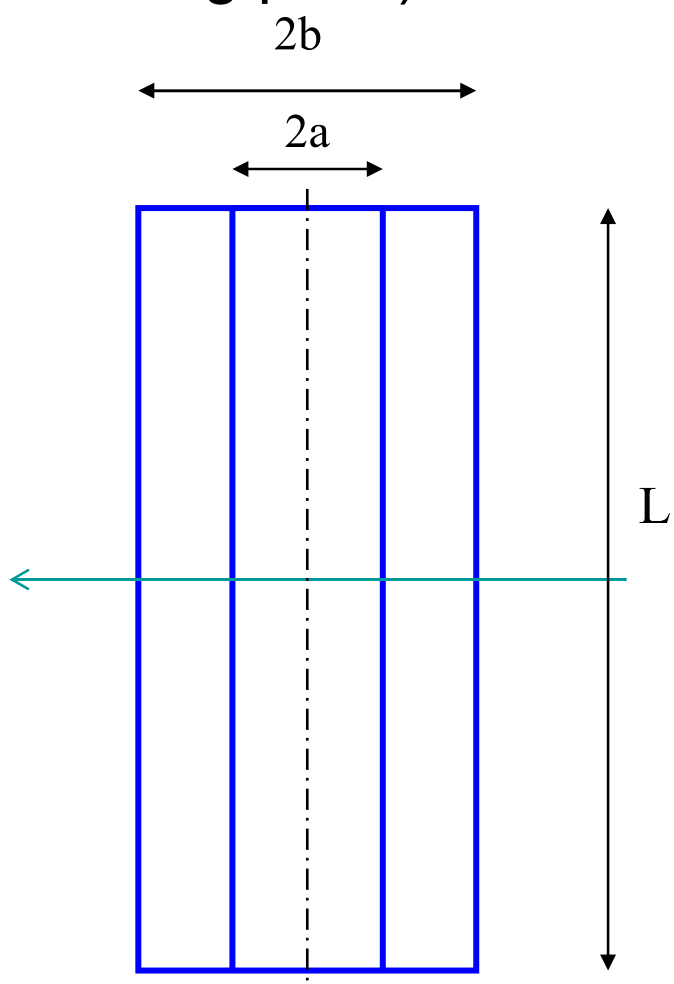


Coaxial Half-wave Resonator

Power dissipation (ignore losses in the shorting plate)

$$P = V_p^2 \frac{16}{\pi} \frac{R_s}{\eta^2} \frac{\lambda}{b} \frac{1+1/\rho_0}{\ln^2 \rho_0}$$

$$P \propto \frac{R_s}{\eta^2} E^2 \beta \lambda^2$$

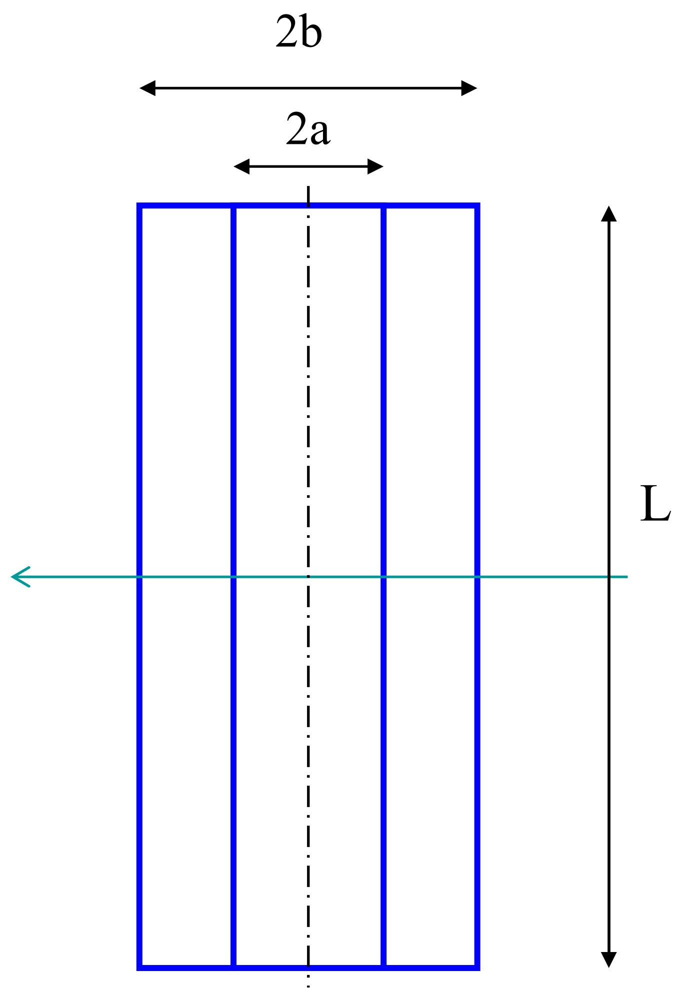


Coaxial Half-wave Resonator

Energy content

$$U = V_p^2 \frac{\pi \varepsilon_0}{4} \lambda \frac{1}{\ln(1/\rho_0)}$$

$$U \propto \varepsilon_0 E^2 \beta^2 \lambda^3$$

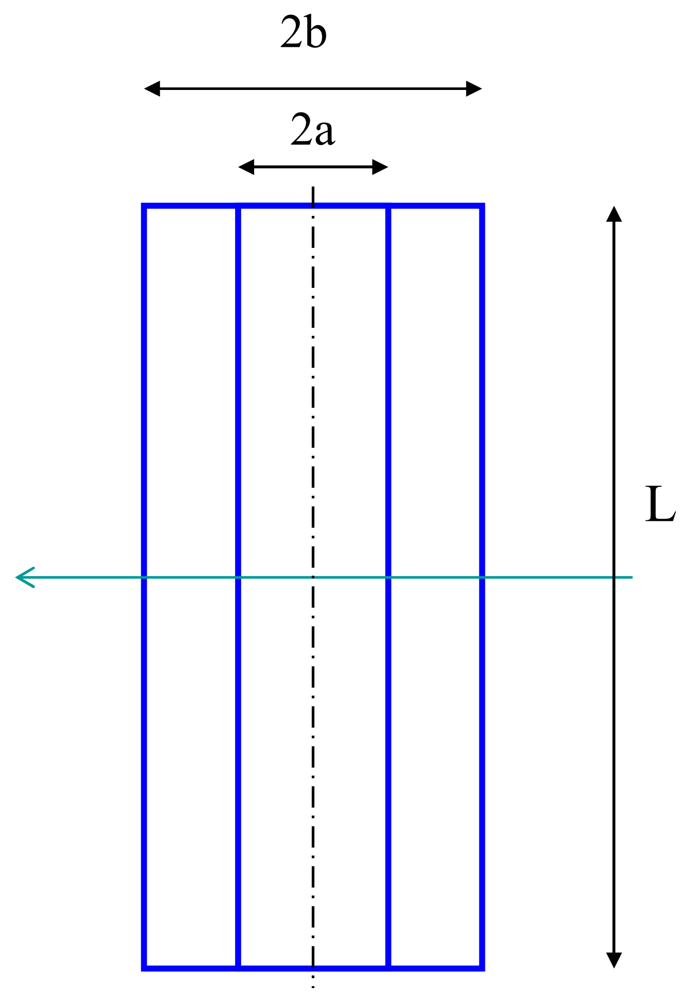


Coaxial Half-wave Resonator

Geometrical factor

$$G = QR_s = 2\pi\,\eta\,\frac{b}{\lambda}\,\frac{\ln(1/\rho_0)}{1+1/\rho_0}$$

$$G \propto \eta\,\beta$$

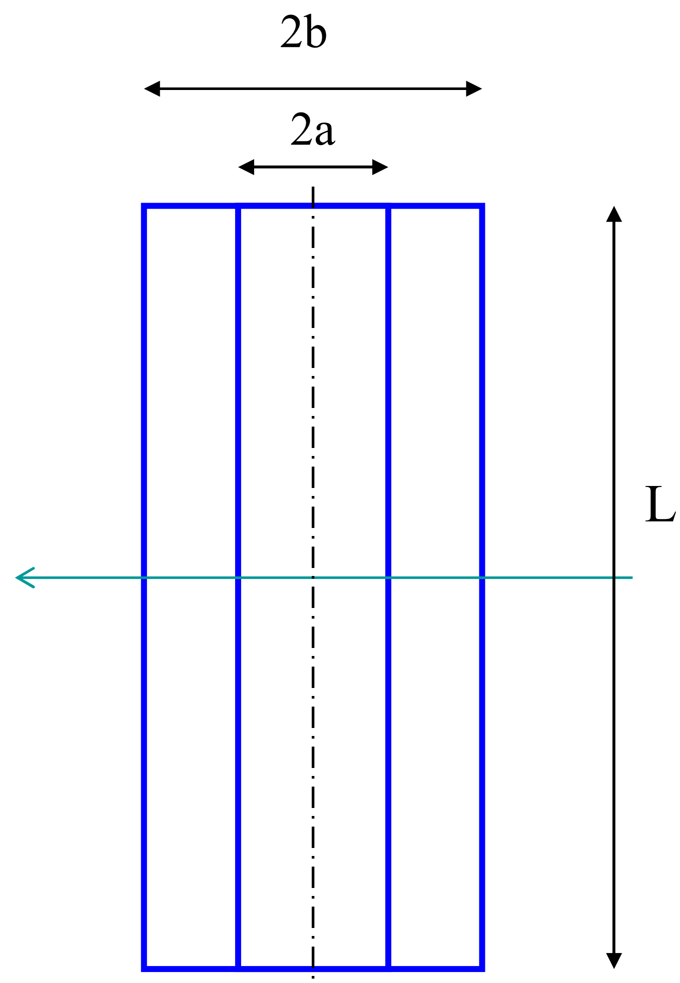


Coaxial Half-wave Resonator

Shunt impedance $\left(4V_p^2 / P\right)$

$$R_{sh} = \frac{\eta^2}{R_s} \frac{16}{\pi} \frac{b}{\lambda} \frac{\ln^2 \rho_0}{1+1/\rho_0}$$

$$R_{sh}\ R_s \propto \eta^2\ \beta$$

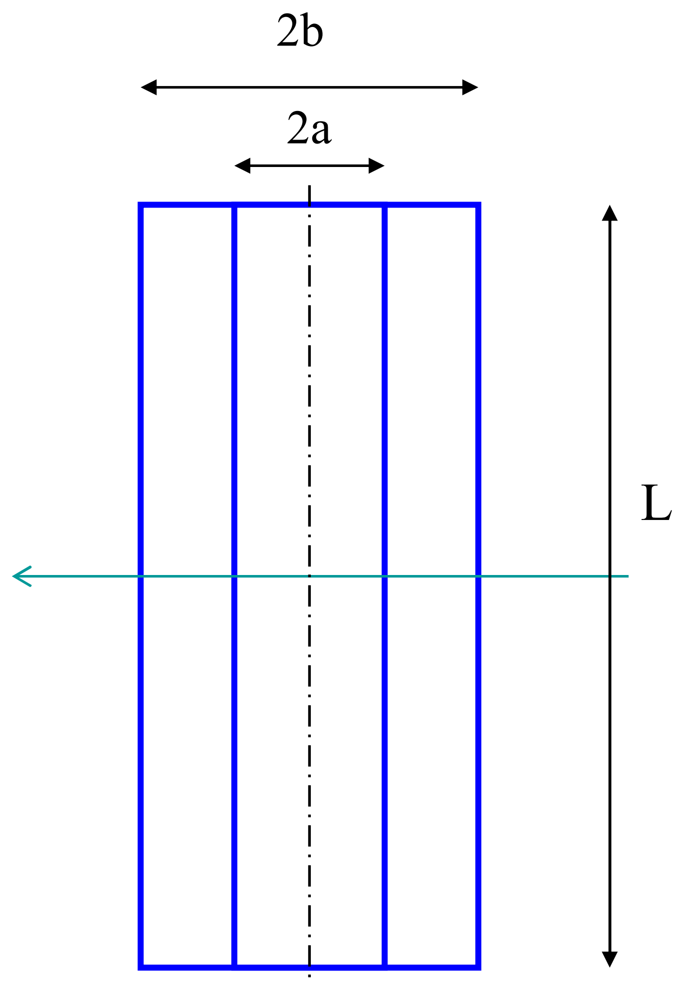


Coaxial Half-wave Resonator

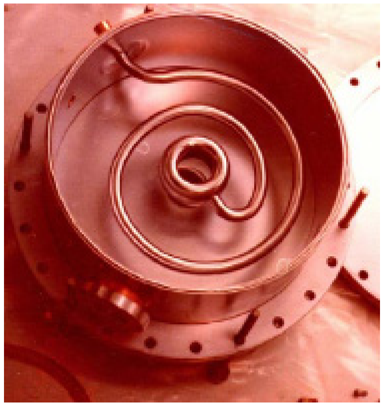
R/Q

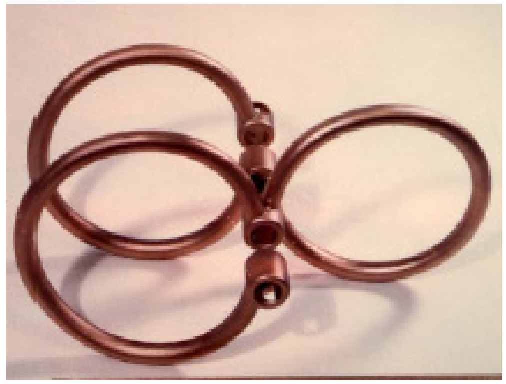
$$\frac{R_{sh}}{Q} = \frac{8}{\pi^2} \eta \ln\left(1/\rho_0\right)$$

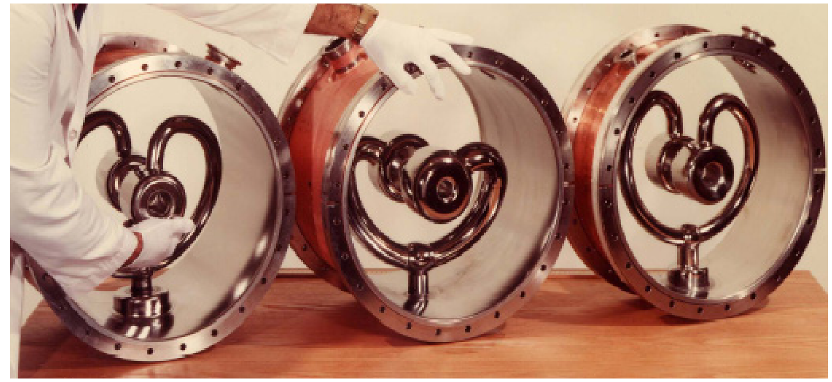
$$\frac{R_{sh}}{Q} \propto \eta$$

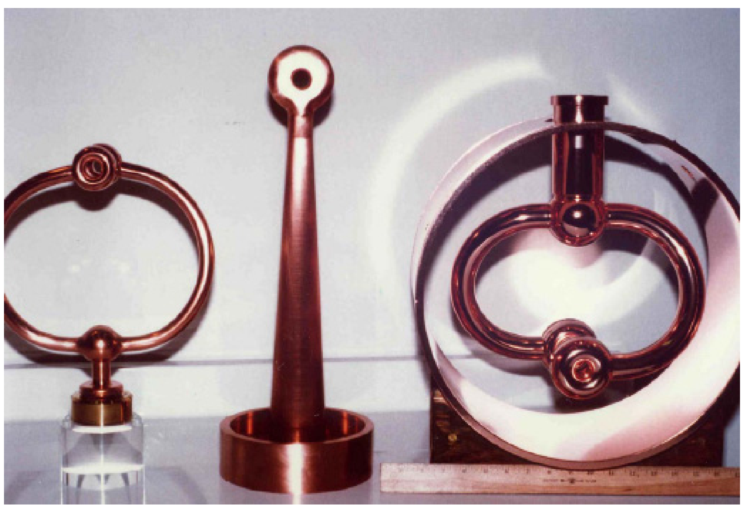


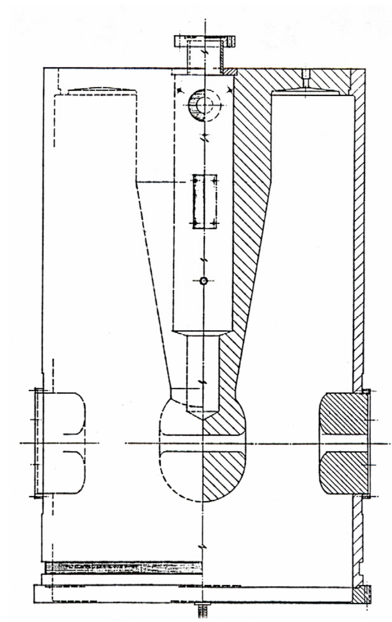
Some Real Geometries (λ/4)



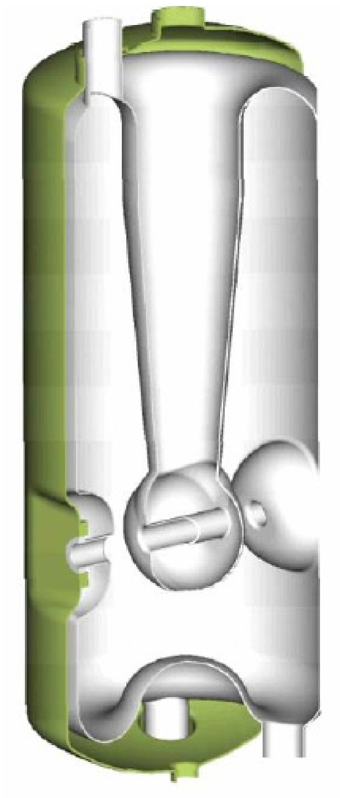


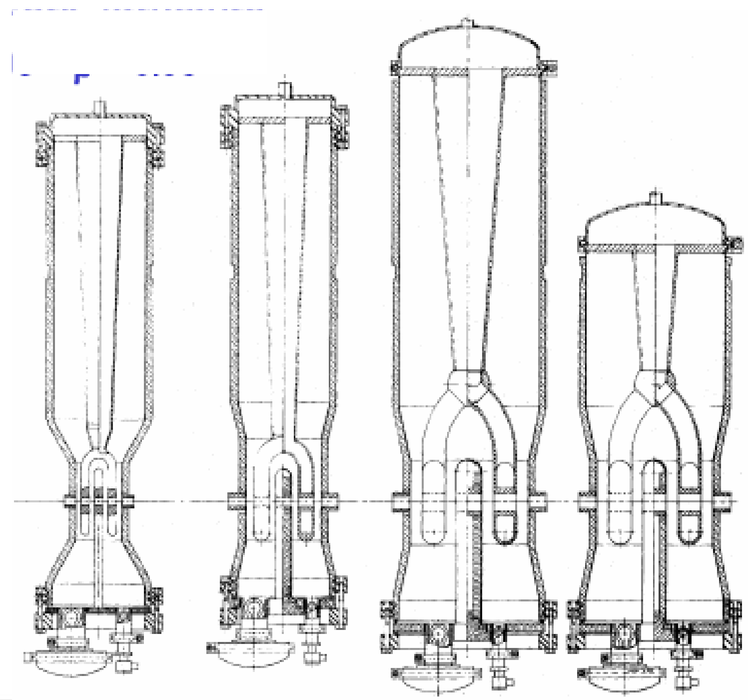




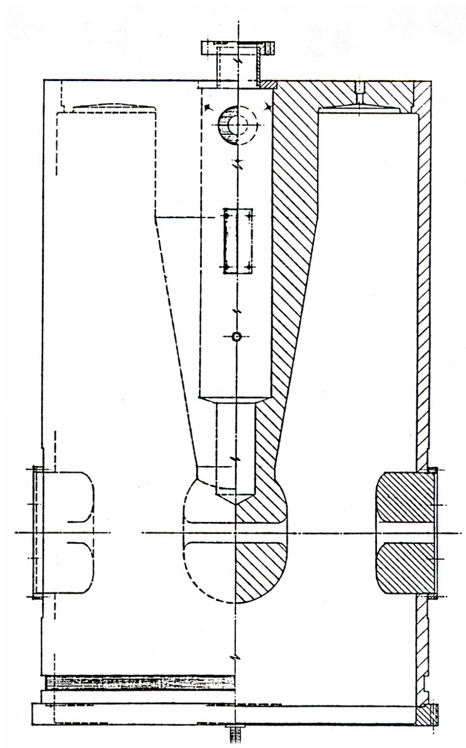


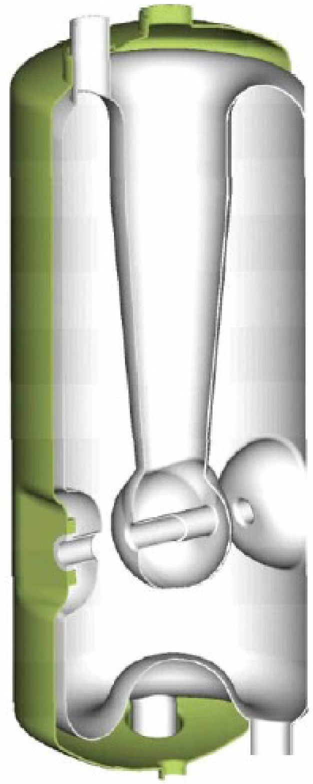
Some Real Geometries (λ/4)

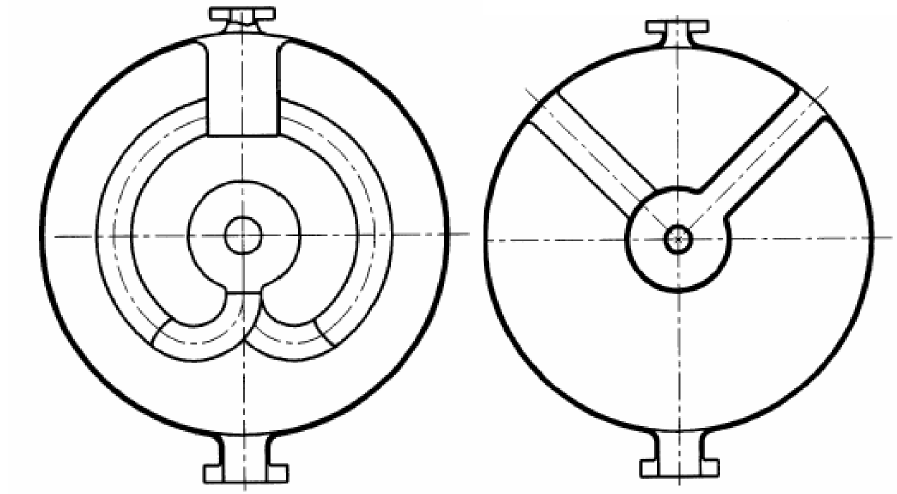


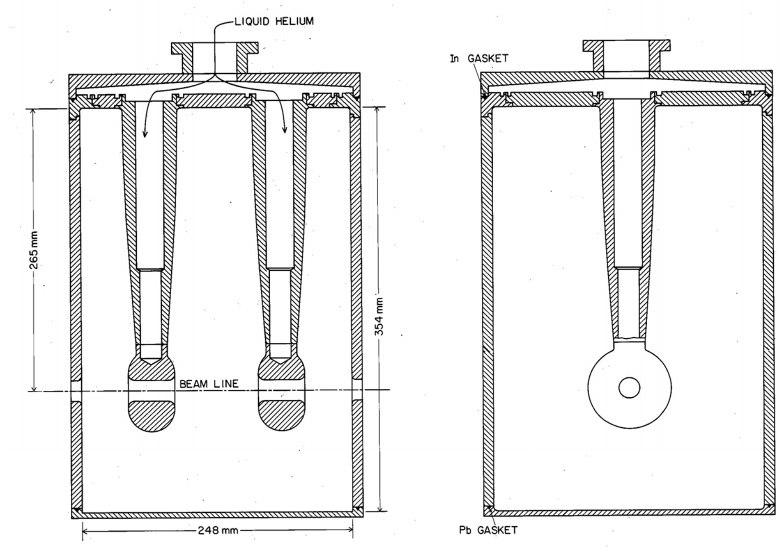


$\lambda/4$ Resonant Lines

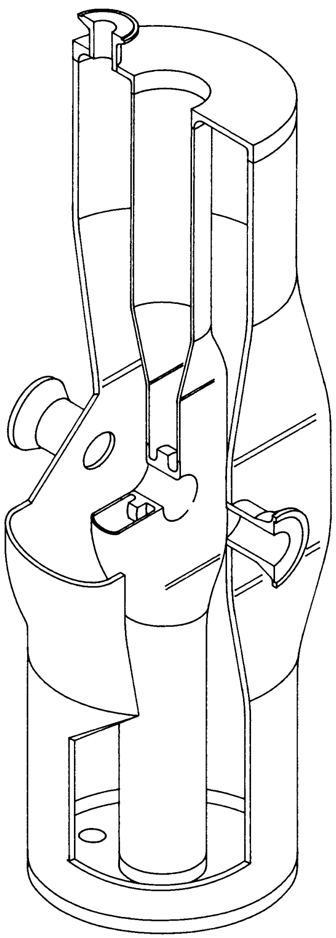


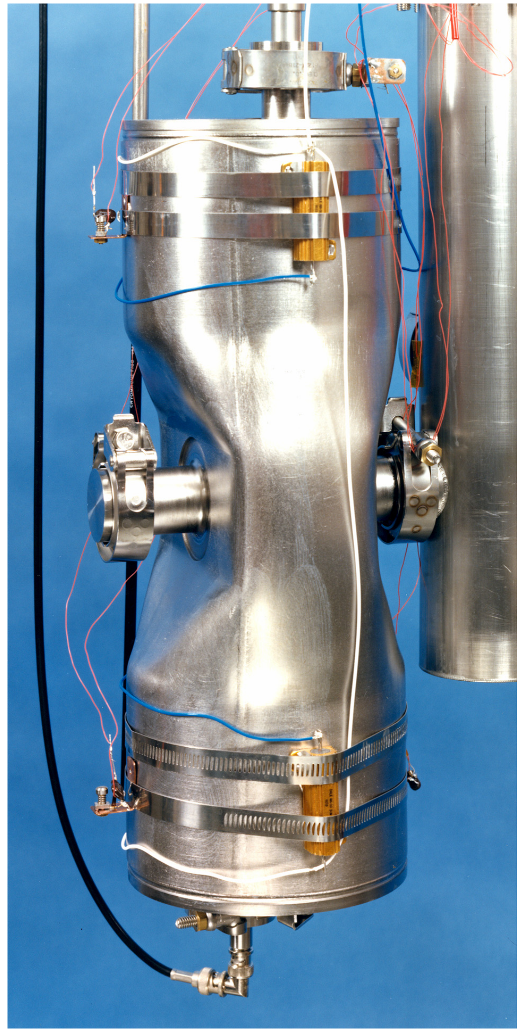


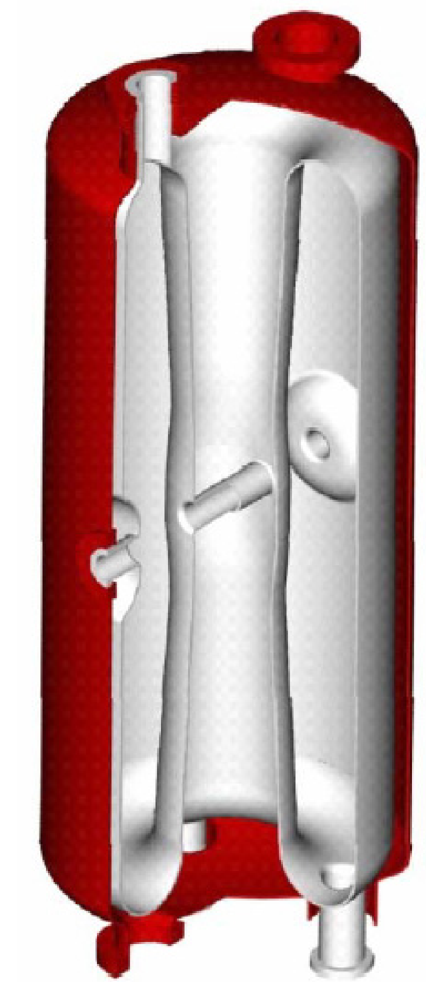




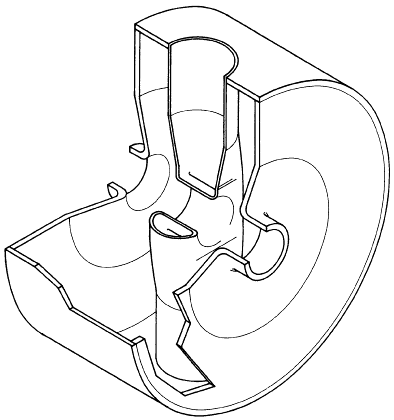
λ/2 Resonant Lines

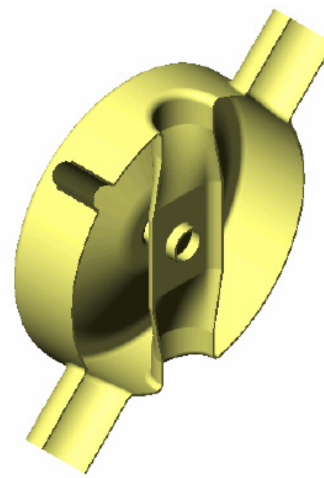


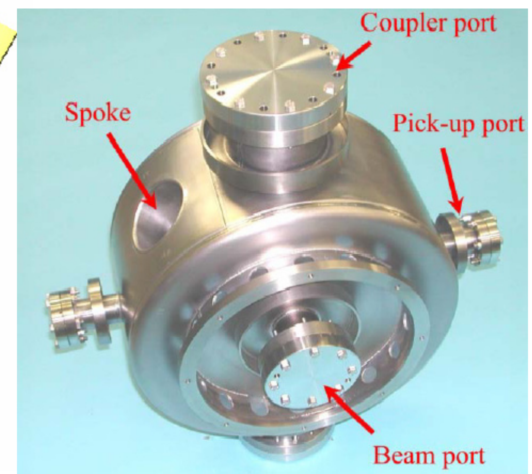




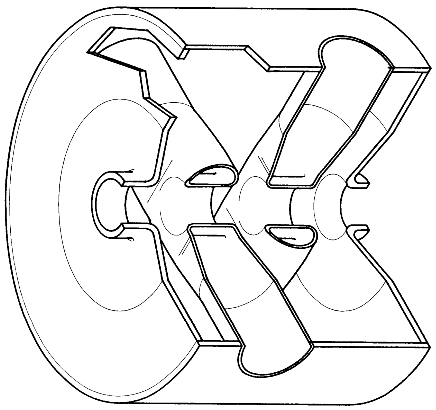
λ/2 Resonant Lines – Single-Spoke

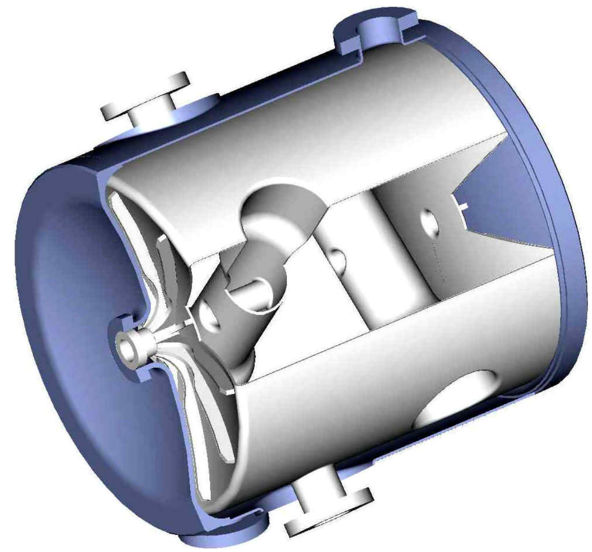


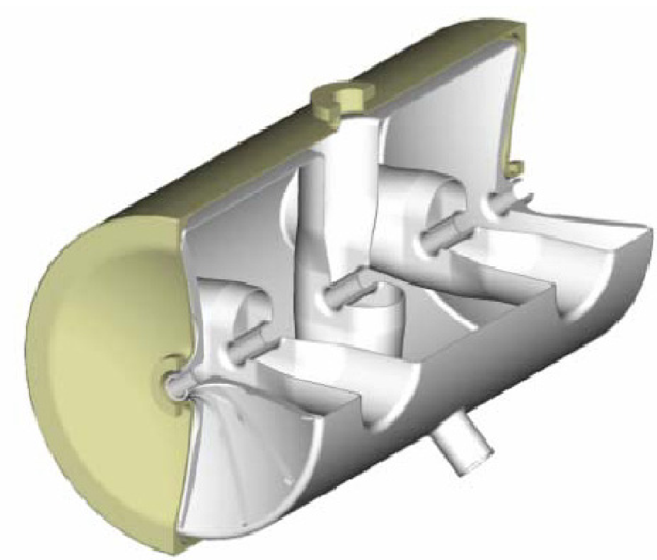




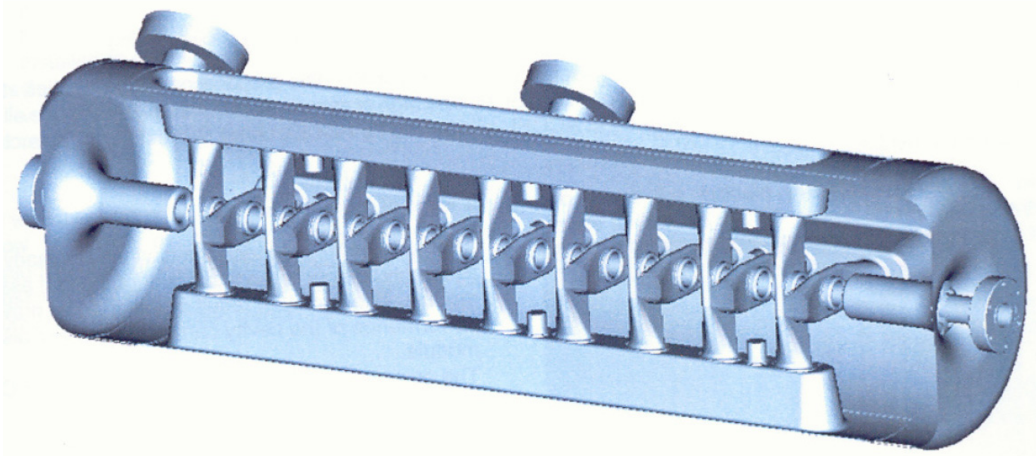
λ/2 Resonant Lines – Double and Triple-Spoke

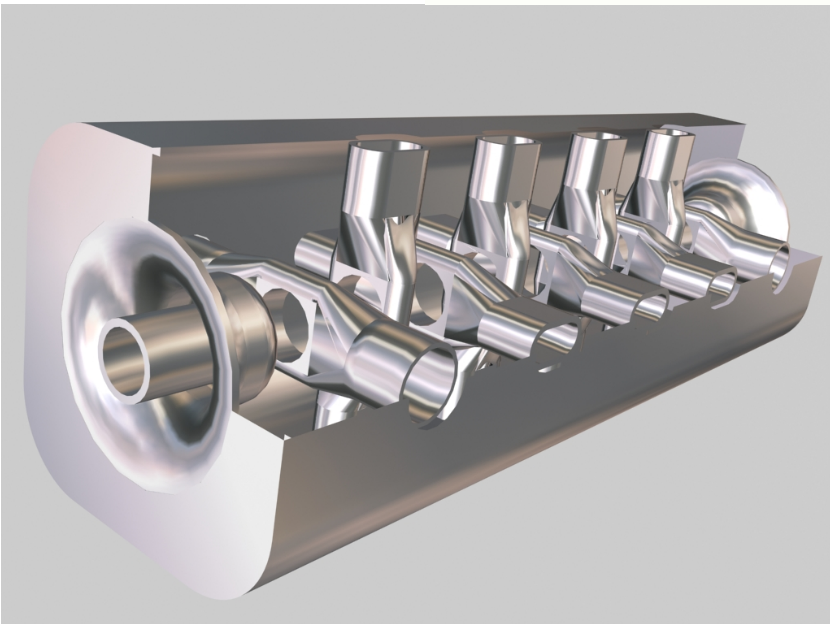


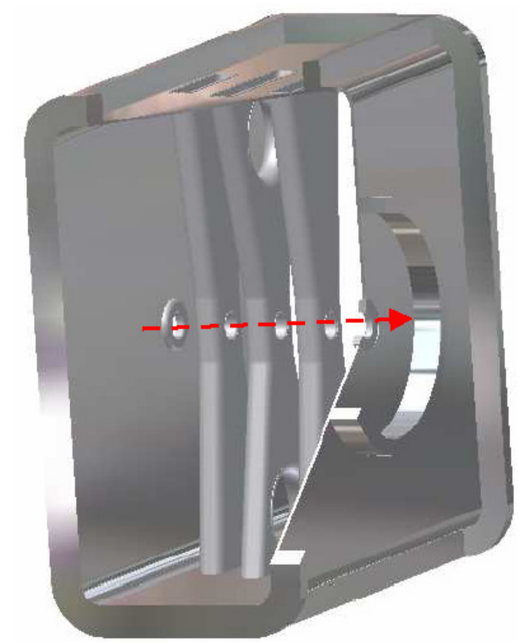




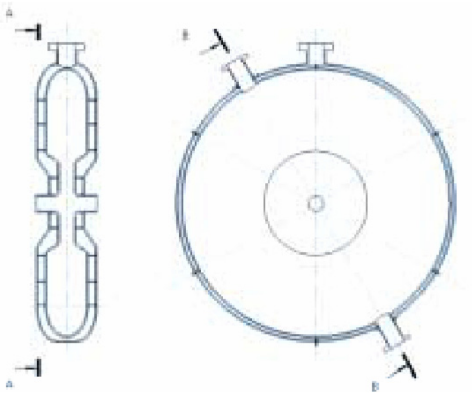
λ/2 Resonant Lines – Multi-Spoke







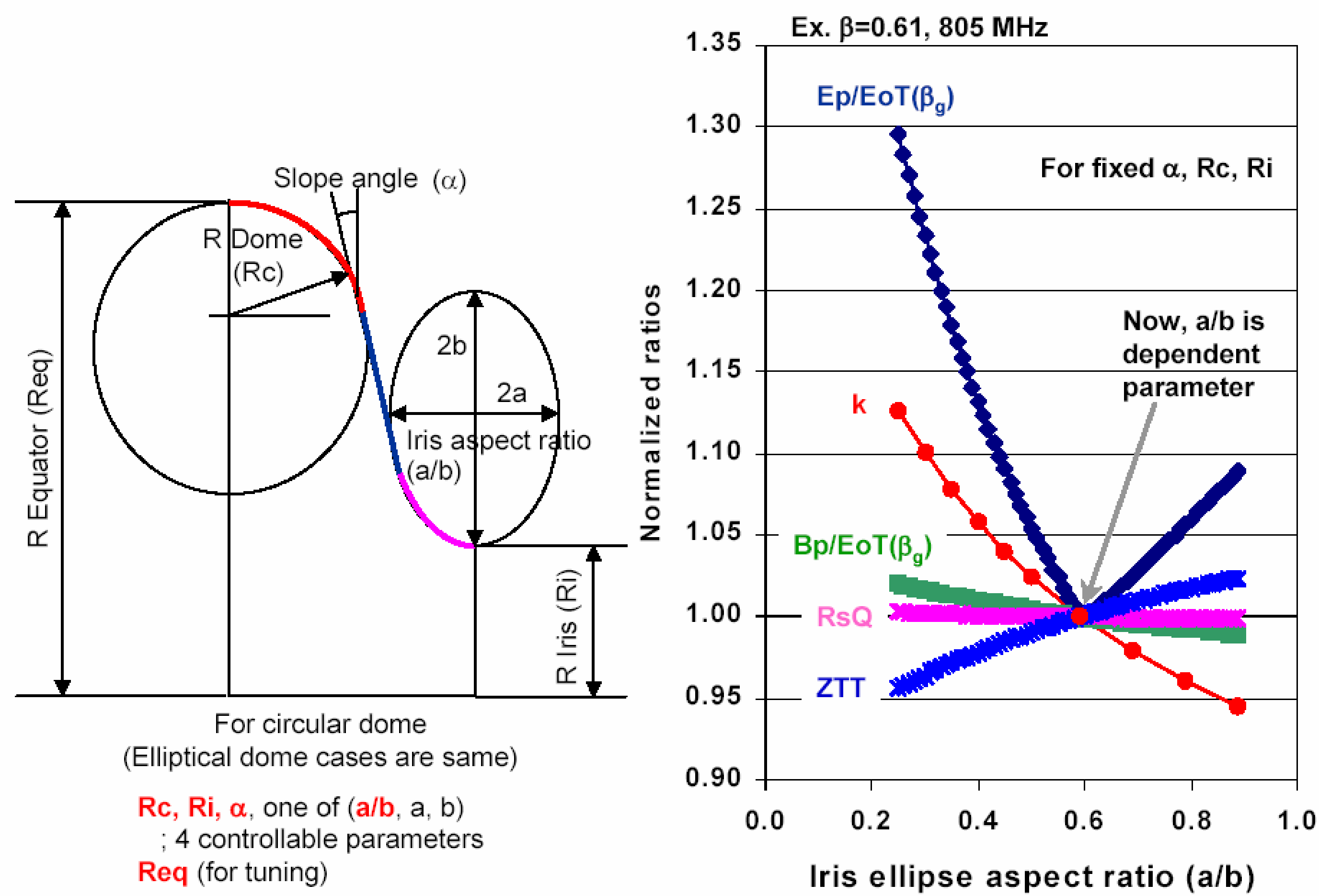
TM Modes



Design Considerations

- Minimize the **peak surface fields**
  Bp; approaches to theoretical limit (190 mT)
  ← high RRR, defect control, better surface treatment (~170 mT)
  Ep; fields exceed 80 MV/m ← improved surface cleaning tech.
- Reasonable **Inter-cell coupling** between cells in Elliptical cavity
- Spoke cavity intrinsically has big coupling constant
- Provide required **external Q**
- In CW, **higher shunt impedance** (mainly determined by the cavity type)
- Reasonable **mechanical stiffness**
  common; reasonable tuning force, mechanical stability under vacuum pressure (test~2 atm), stable against microphonics
  pulsed; affordable dynamic Lorentz force detuning
- Safe from **Multipacting**
- Verify **HOM** and related issues
- **Coupled field problems** are common between RF, mechanical, thermal..
  → strong interfaces are needed

RF Geometry Optimization (elliptical cavity)

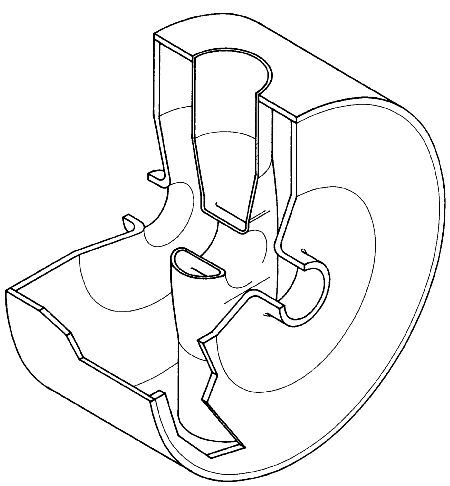


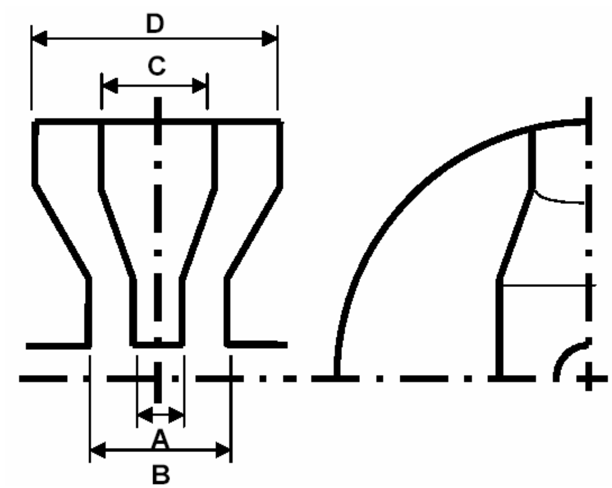
Elliptical cell geometry and dependencies of RF parameters on the ellipse aspect ratio (a/b) at the fixed slope angle, dome radius and bore radius.

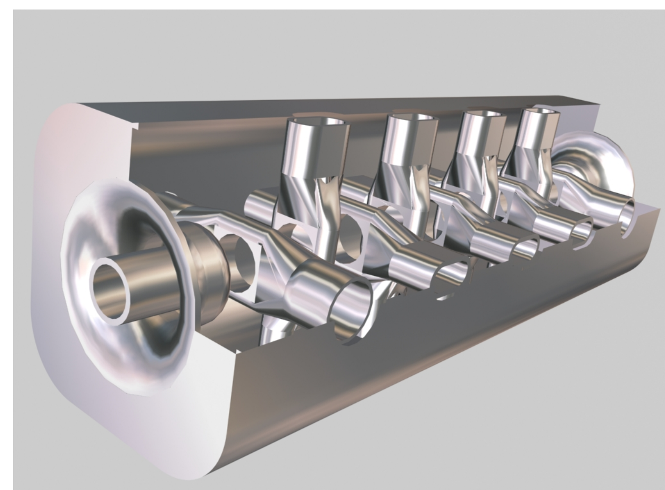
RF Geometry Optimization (Spoke Cavity)

- There have been extensive efforts for design optimization especially to reduce the ratios of Ep/Eacc and Bp/Eacc.
  - Controlling A/B (Ep/Eacc) and C/D (Bp/Eacc) → Shape optimization
  - Flat contacting surface at spoke base will help in another minimization of Bp/Eacc
  - For these cavities:

    Calculations agree well→ Ep/Eacc~3, Bp/Eacc~(7~8) mT/(MV/m),
    though it is tricky to obtain precise surface field information from the 3D simulation.
    Intrinsically have very strong RF coupling in multi-gap cavity.
    Have rigid nature against static and dynamic vibrations.
    Beta dependency is quite small.
    Diameter~half of elliptical cavity.







Velocity Acceptance

- **Energy gain** $$\Delta W = q\ V\ T(x)\ \ \Phi(x)\ \ \cos\varphi$$

$$x = \frac{\beta\lambda}{2l}$$

$T(x)$ **Transit time factor for single cell**

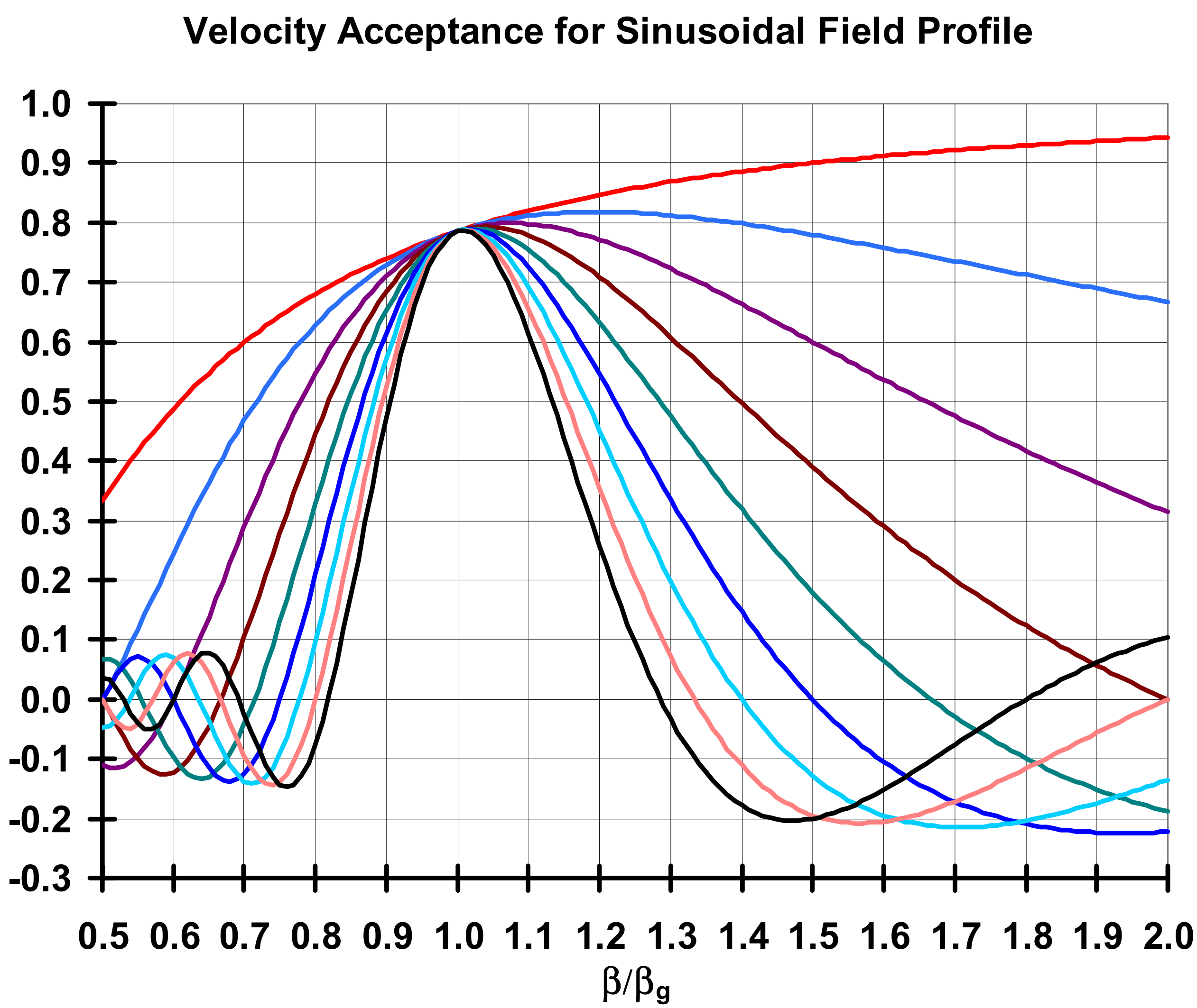
Depends on field profile in cell

$\Phi(x)$ **Phasing factor in multicell cavities**

Depends on cell spacing and field amplitude in cells

Does not depend on field profile in cells (assumed to be identical)

Velocity Acceptance

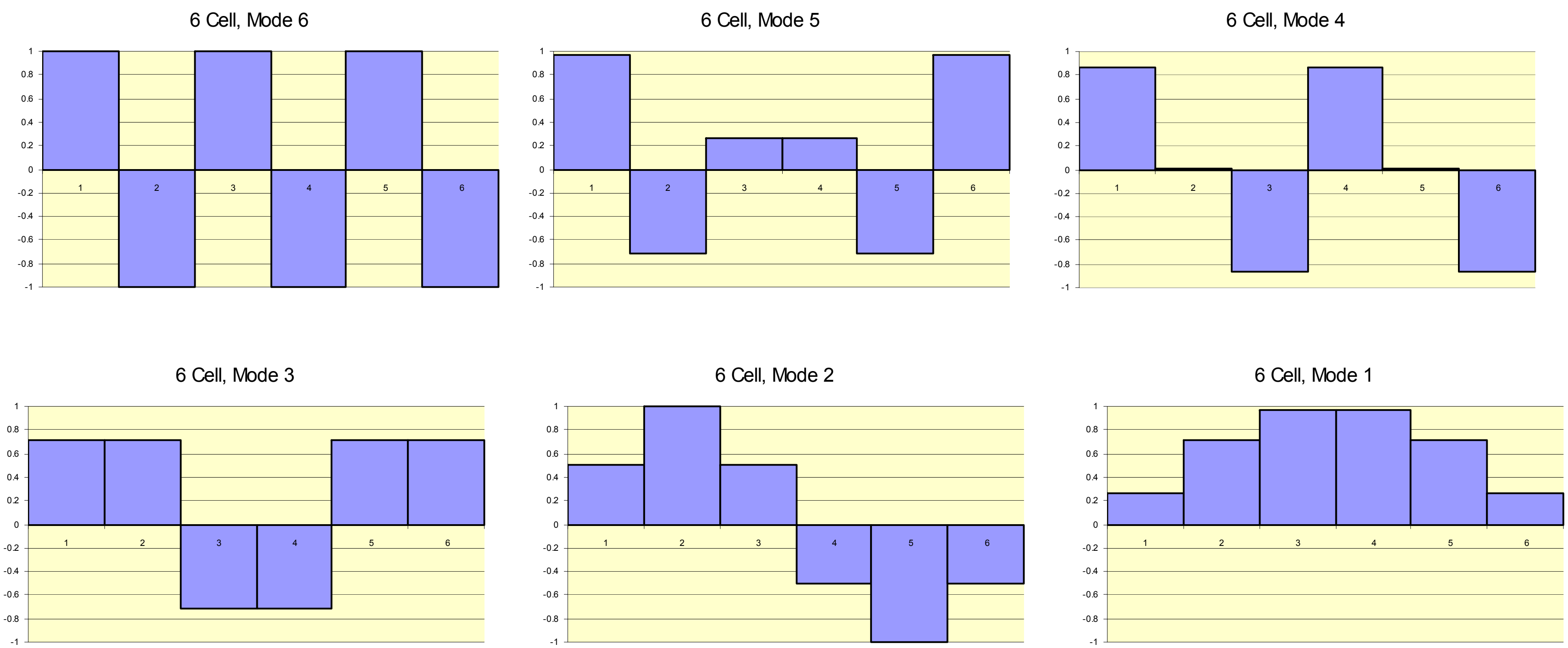


Voltage in Cells

Voltage in j^th cell

$$V_j^M = \sin\left(\pi M \frac{(2j-1)}{2N}\right)$$

N: Number of cells, M: Mode number



Phasing Factor

For fundamental (π) mode: $$\Phi(x) = \frac{1}{\cos\left(\frac{\pi}{2x}\right)} \begin{cases} (-1)^{n+1}\sin\left(\frac{N\pi}{2x}\right), & N = 2n \\ (-1)^{n}\cos\left(\frac{N\pi}{2x}\right), & N = 2n+1 \end{cases}$$

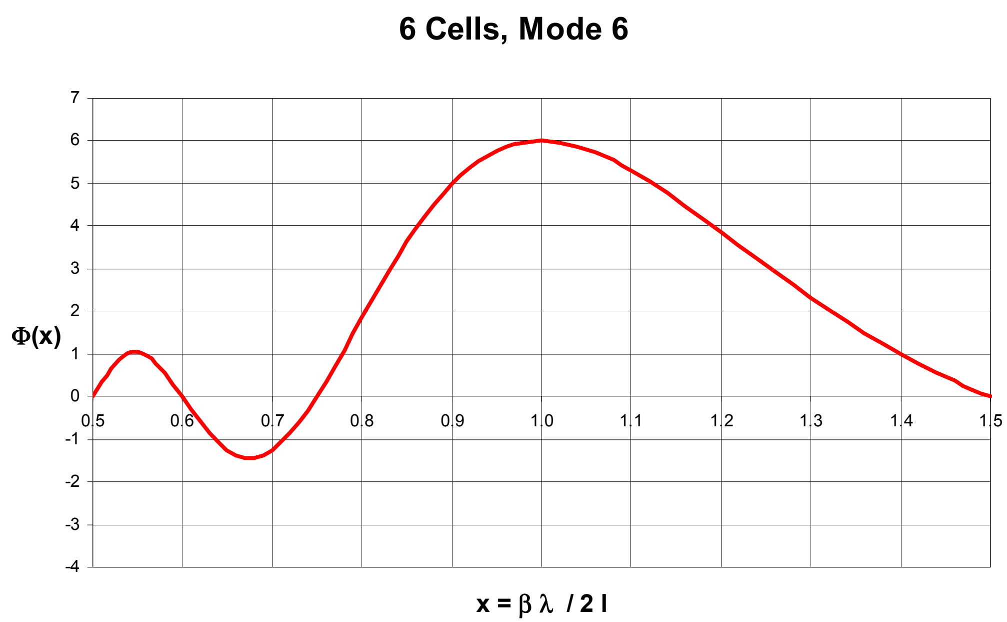
For all modes:

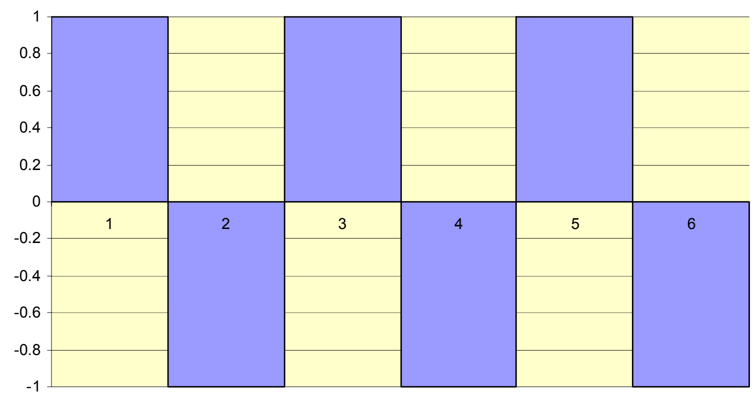
$$\Phi(x) = \frac{1}{2}\left(\frac{\sin\left[\frac{N\pi}{2}\left(\frac{M}{N} - \frac{1}{x}\right)\right]}{\sin\left[\frac{\pi}{2}\left(\frac{M}{N} - \frac{1}{x}\right)\right]} + (-1)^{M+1} \frac{\sin\left[\frac{N\pi}{2}\left(\frac{M}{N} + \frac{1}{x}\right)\right]}{\sin\left[\frac{\pi}{2}\left(\frac{M}{N} + \frac{1}{x}\right)\right]} \right)$$

If M=N, recover previous formula

If x=1 $$\Phi(x) = N\delta_{MN}$$

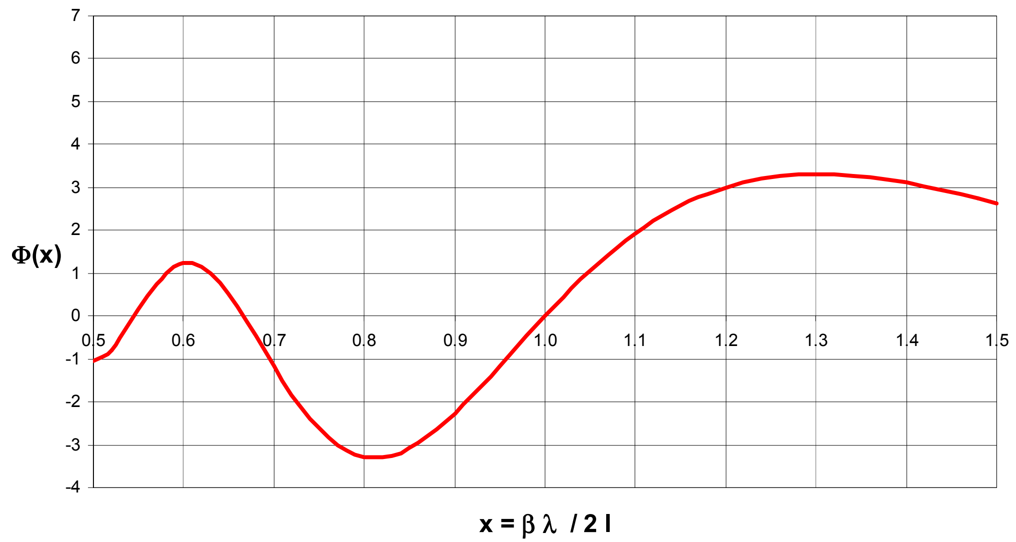
Phasing Factor

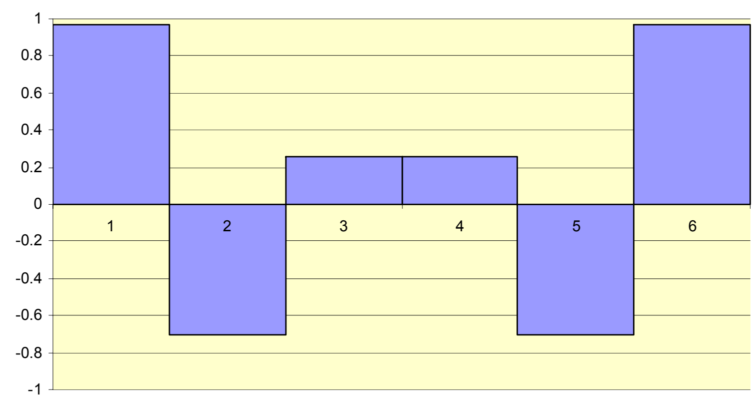




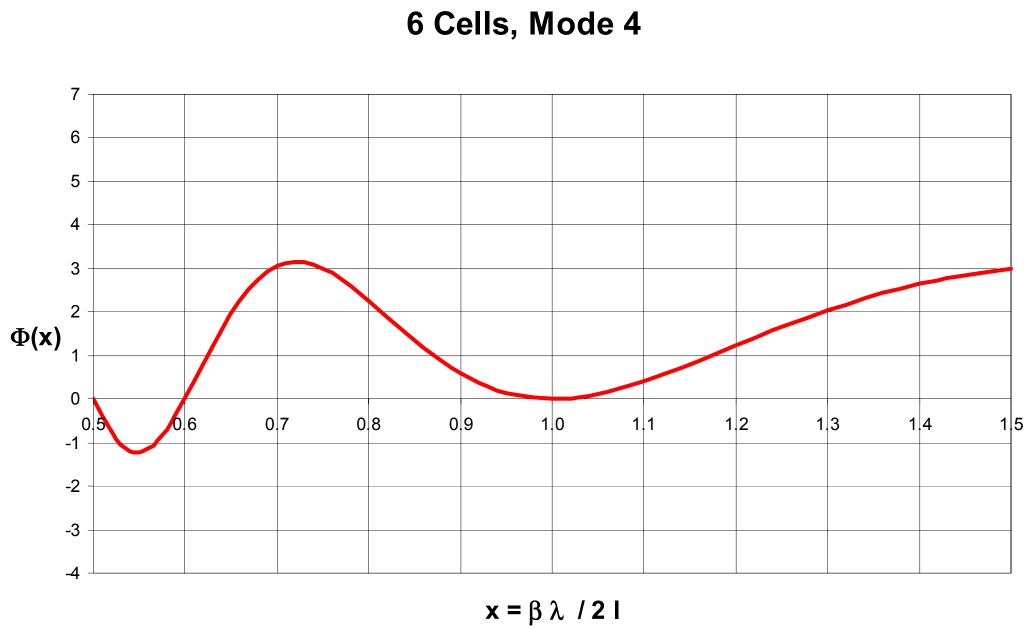
Phasing Factor

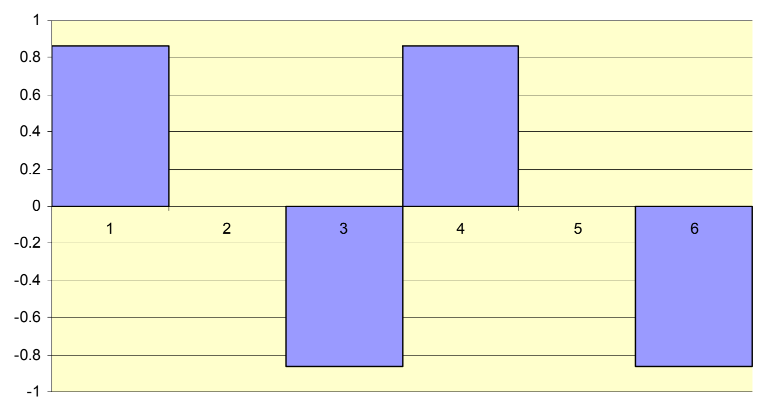
6 Cells, Mode 5





Phasing Factor





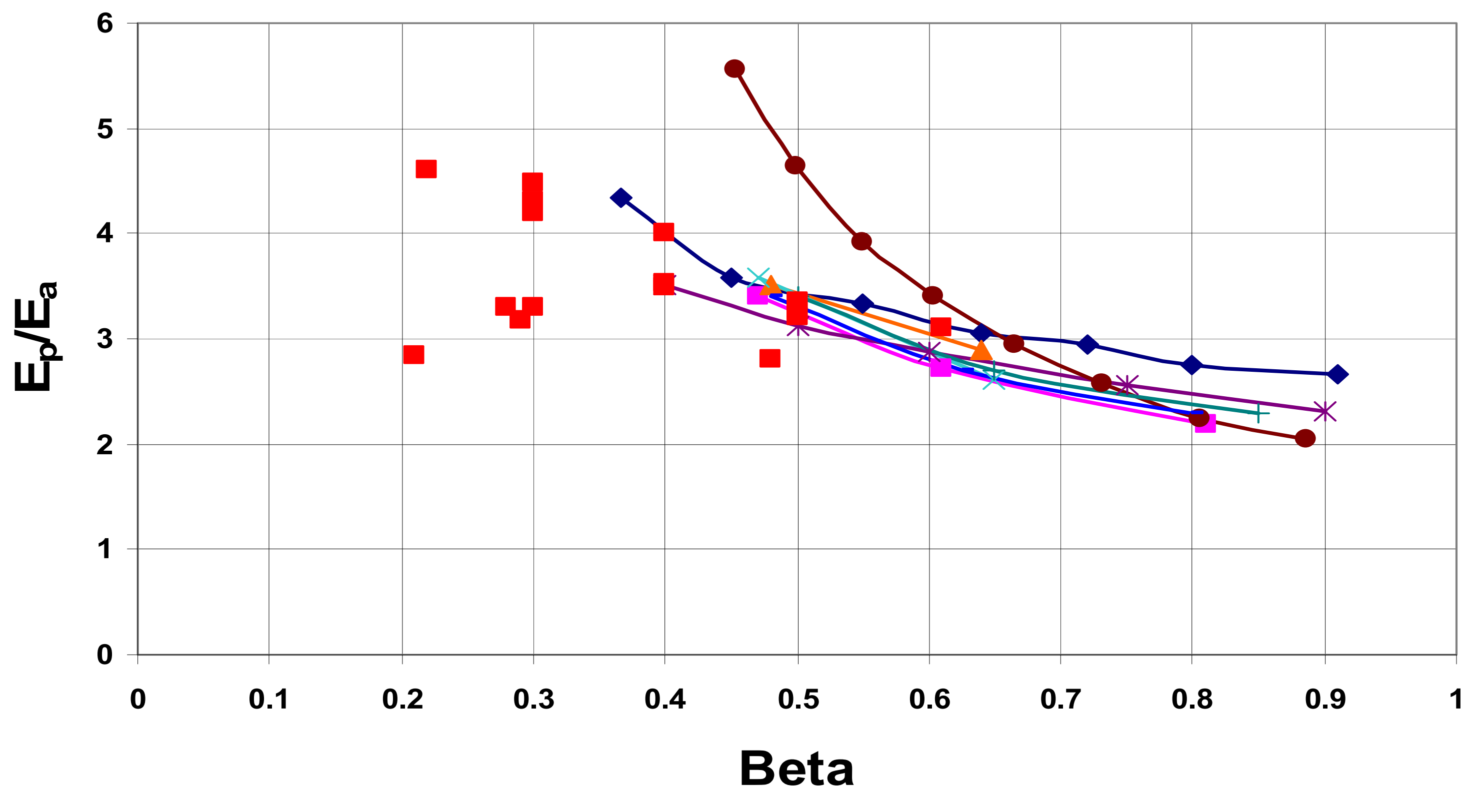
Surface Electric Field

- TM_{010} elliptical structures
  - $E_p/E_a \sim 2$ for $\beta = 1$
  - Increases slowly as β decreases

- $\lambda/2$ structures:
  - Sensitive to geometrical design
  - Electrostatic model of an "shaped geometry" gives $E_p/E_a \sim 3.3$, independent of β

Surface Electric Field

- **Lines: Elliptical** **Squares: Spoke**



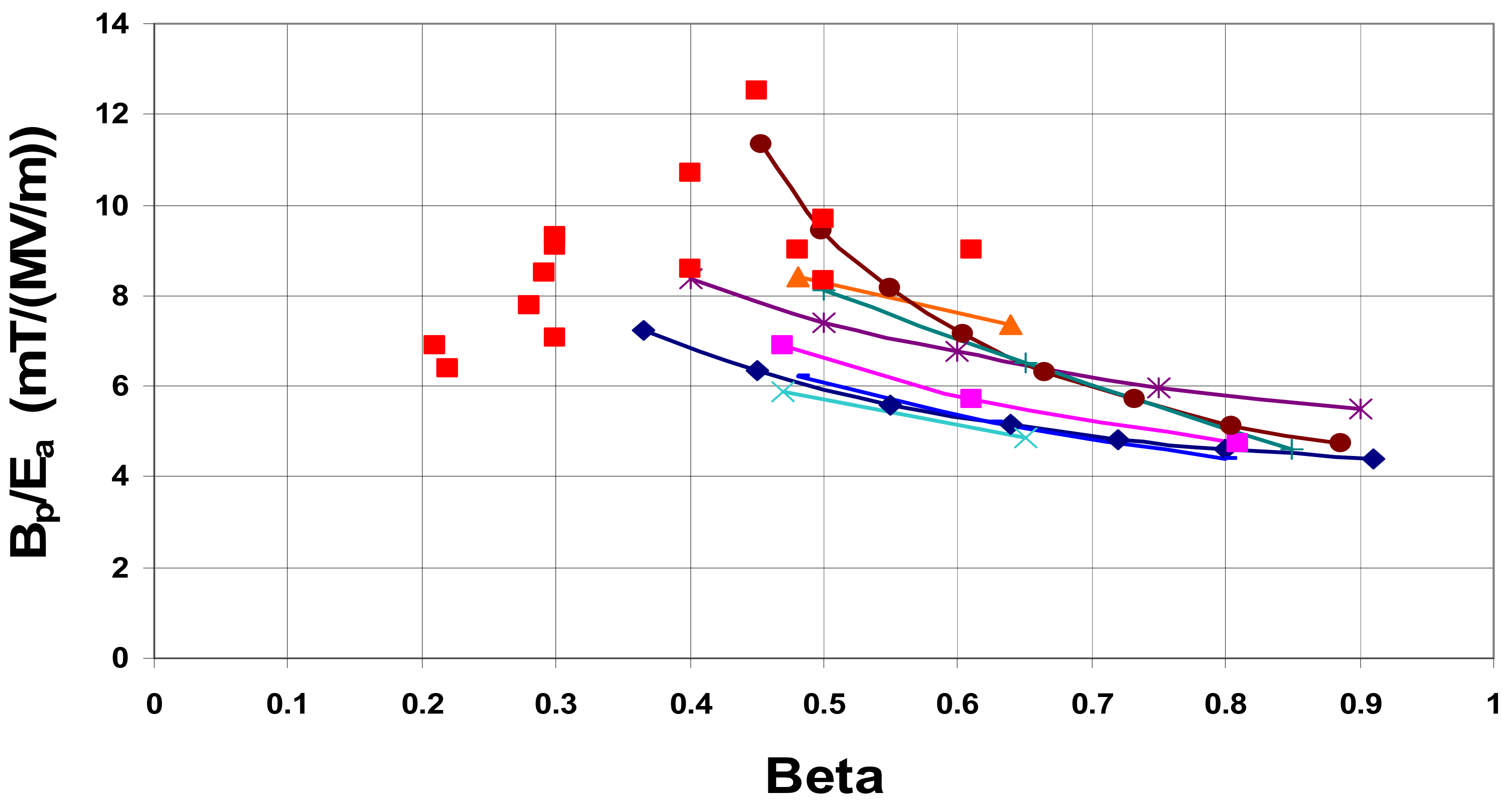
Surface Magnetic Field

- **TM_{010} elliptical cavities:**
  - **B/E_a ~ 4 mT/(MV/m) for β=1**
  - **Increases slowly as β decreases**

- **λ/2 structures:**
  - **Sensitive to geometrical design**
  - **Transmission line model gives B/E_a ~ 8 mT/(MV/m), independent of β**

Surface Magnetic Field

- **Lines: Elliptical** **Squares: Spoke**

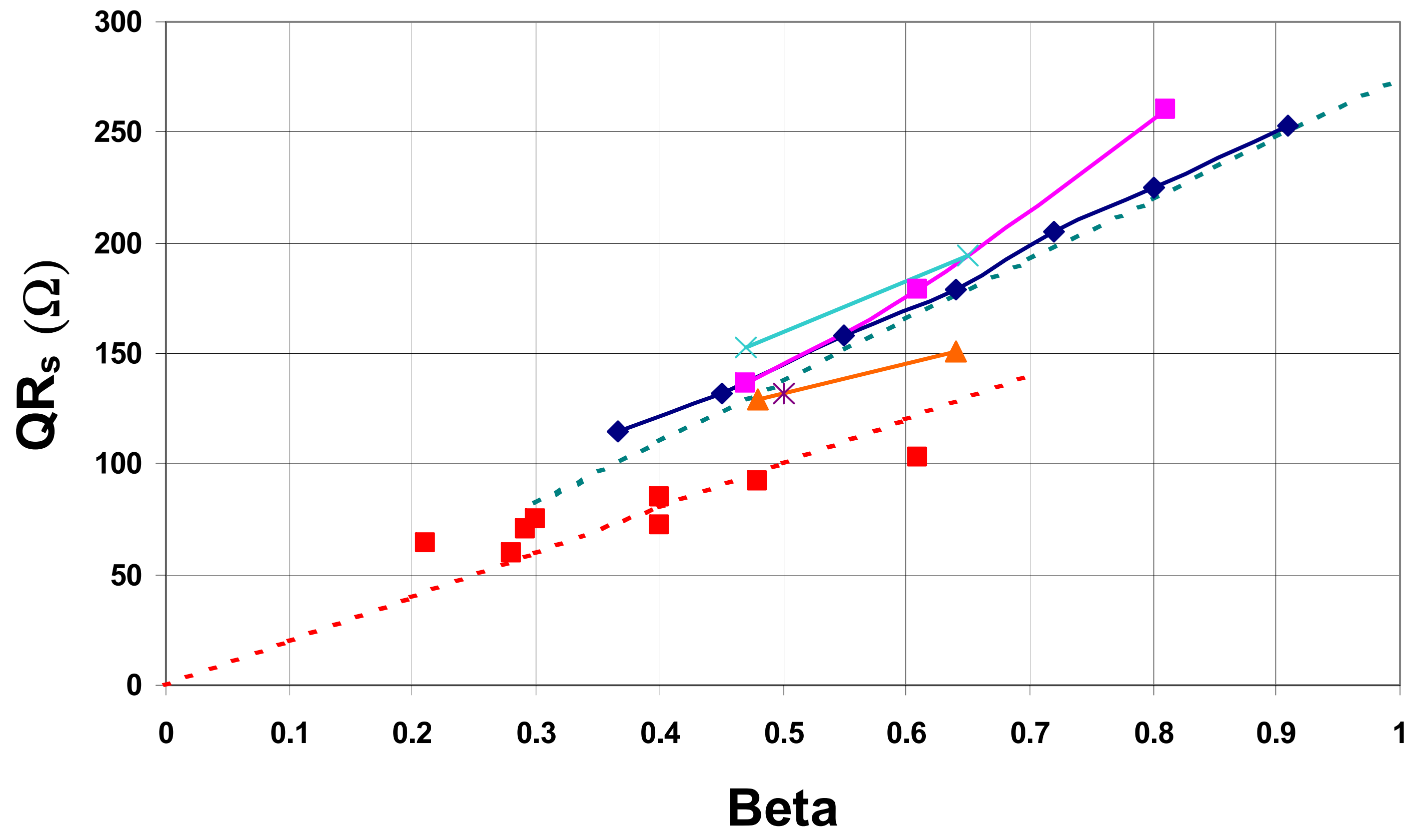


Geometrical Factor (QR_s)

- TM_{010} elliptical cavities:
  - Simple scaling: $QR_s \sim 275\,\beta\ (\Omega)$
- $\lambda/2$ structures:
  - Transmission line model: $QR_s \sim 200\,\beta\ (\Omega)$

Geometrical Factor (QR_s)

- Lines: Elliptical Squares: Spoke



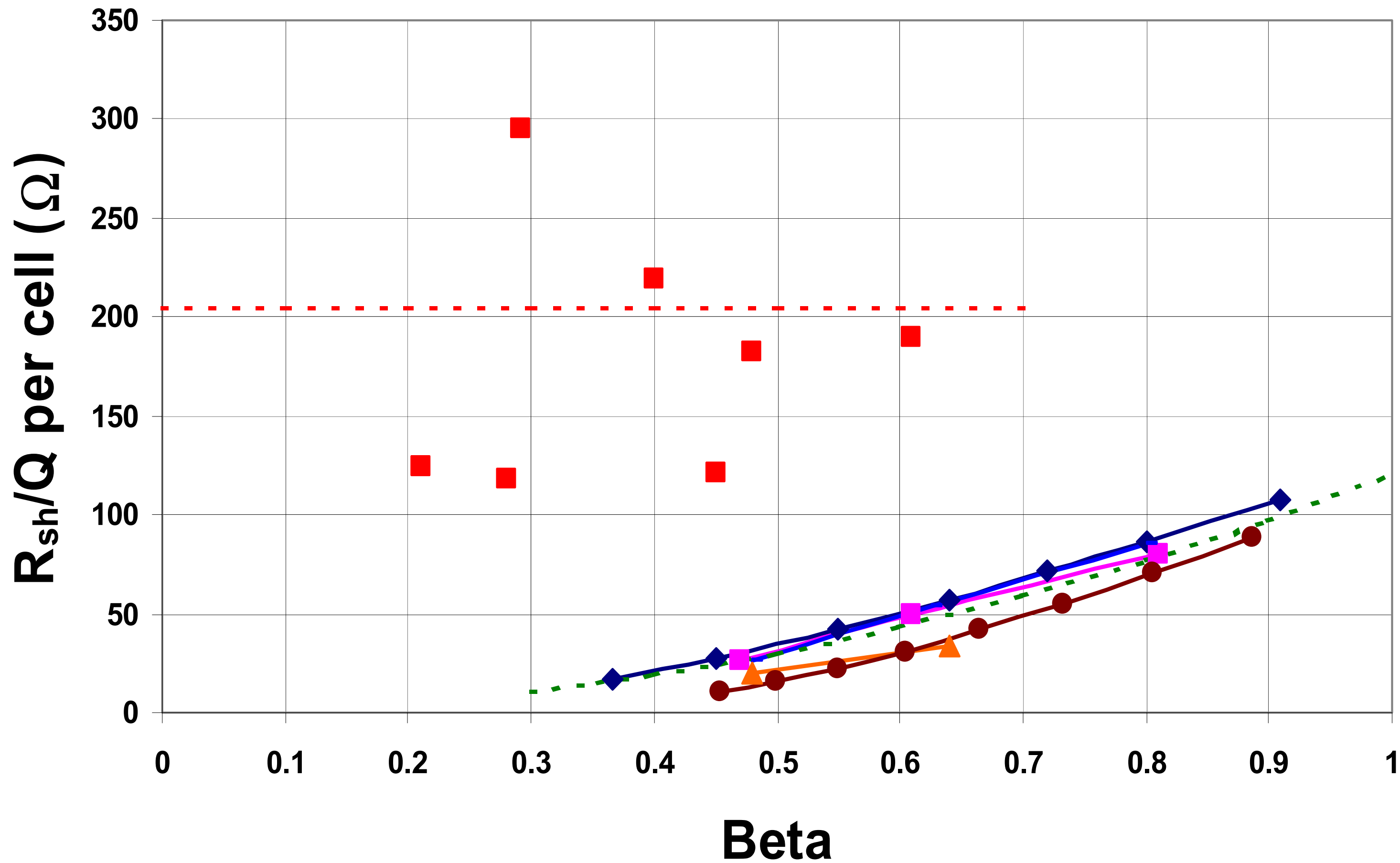
R_{sh}/Q per Cell or Loading Element

- $R_{sh} = V^2/P$
- TM_{010} elliptical cavities:
  - Simple-minded argument, ignoring effect of beam line aperture, gives: $R_{sh}/Q \propto \beta$
  - When cavity length becomes comparable to beam line aperture : $R_{sh}/Q \propto \beta^2$
  - $R_{sh}/Q \approx 120\ \beta^2 \quad (\Omega)$

- $\lambda/2$ structures:
  - Transmission line model gives: $R_{sh}/Q \approx 205\ \Omega$
  - Independent of β

R_{sh}/Q per Cell or Loading Element

Lines: Elliptical **Squares: Spoke**



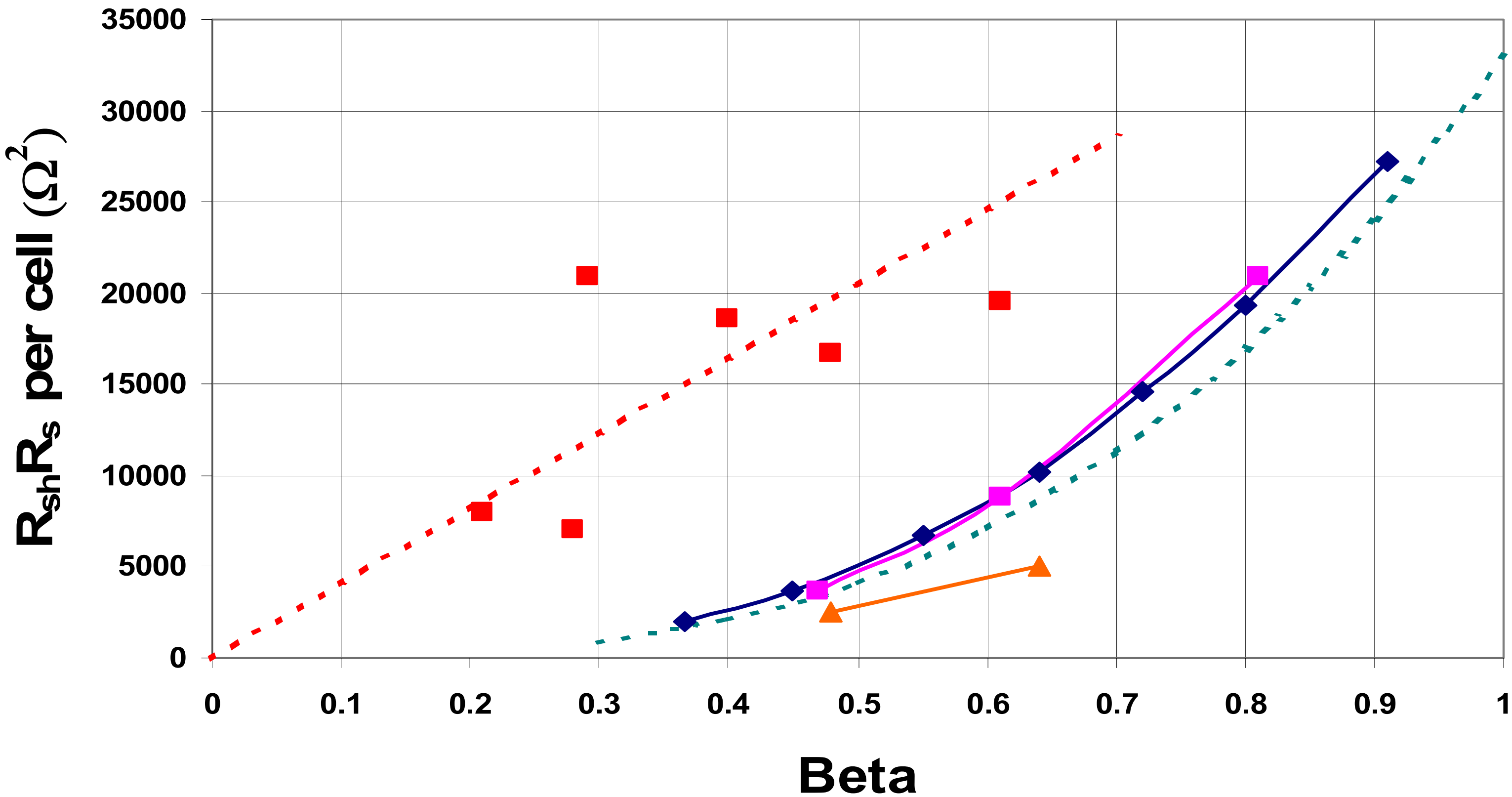
Shunt Impedance R_{sh} ($R_{sh}/Q \quad QR_s$ per Cell or Loading Element)

- **TM_{010} elliptical cavities:**
  - $R_{sh} R_s \sim 33000\ \beta^3 \quad (\Omega^2)$

- **$\lambda/2$ structures:**
  - $R_{sh} R_s \sim 40000\ \beta \quad (\Omega^2)$

Shunt Impedance R_{sh}
(R_{sh}/Q QR_s per cell or loading element)

- Lines: Elliptical   Squares: Spoke



Energy Content per Cell or Loading Element

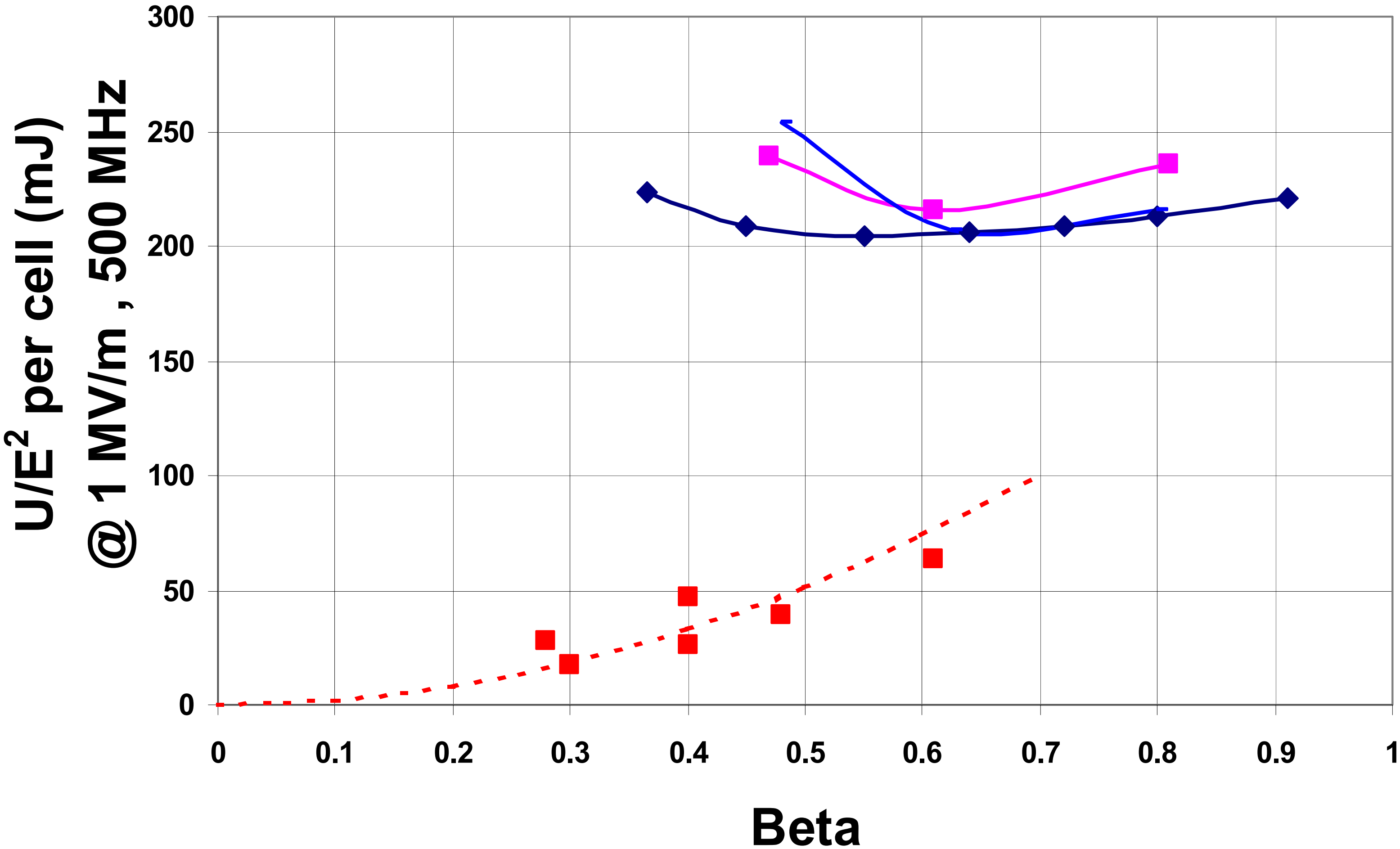
Proportional to $E^2\lambda^3$

At 1 MV/m, normalized to 500 MHz:

- **TM_{010} elliptical cavities:**
  - **Simple-minded model gives $U/E^2 \propto \beta$**
  - **In practice: U/E^2 ~ 200-250 mJ**
  - **Independent of β (seems to increase when β <0.5 – 0.6)**

- **$\lambda/2$ structures:**
  - **Sensitive to geometrical design**
  - **Transmission line model gives $U/E^2 \sim 200\ \beta^2$ (mJ)**

Energy Content per Cell or Loading Element



Size & Cell-to-Cell Coupling

TM_{010} Structures

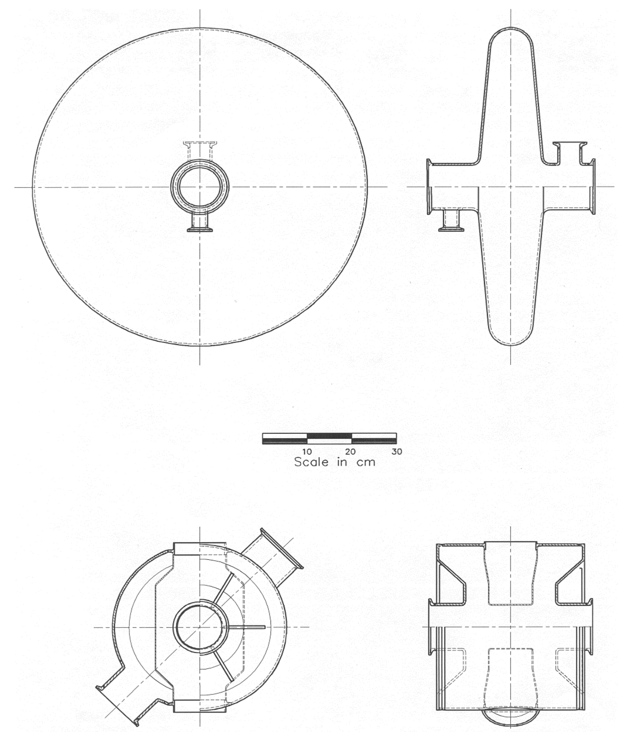
$\varnothing$ ~ 0.88 – 0.92 λ

Coupling ~ 2%

I/2 Structures

$\varnothing$ ~ 0.46 – 0.51 λ

Coupling ~ 20 - 30%



Example : 350 MHz, β= 0.45

Multipacting

- TM_{010} elliptical structures
  - Can reasonably be modeled and predicted/avoided
  - Modeling tools exist

- $\lambda/2$ Structures
  - Much more difficult to model
  - Reliable modeling tools do not exist
  - Multipacting "always" occurs
  - "Never" a show stopper

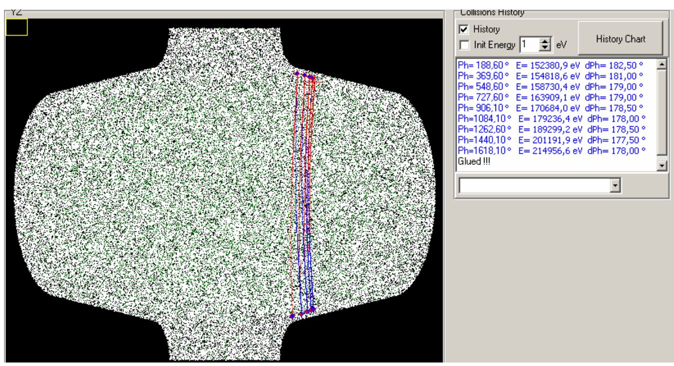
Multipacting

- Resonant electron loading → strongly depends on geometry
- Several 2D simulation codes are available;

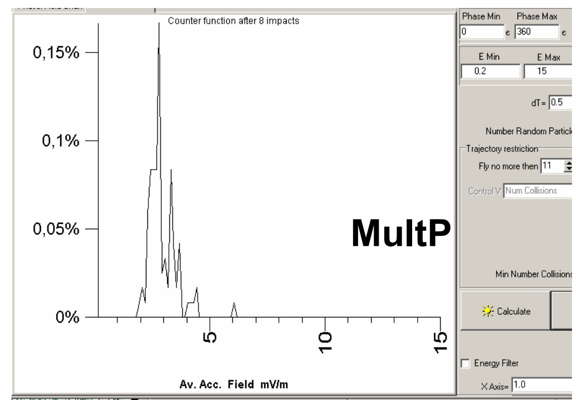
Multipacting condition

1. Closed trajectory
2. insensitive to the initial energy
3. SEY(E)>1 (Physical surface condition) ← lowering thru He processing & better surface cleaning process

- A series validity test tells (Larger) circular dome will help to preventing MP.
  β Lower than b~0.45 cavity is supposed to have more MP problem



Courtesy of L. Kravchuk (INR)



Ex. SNS 0.61 cavity, condition 1&2 are satisfied, but electron energy is too high SEY<<1

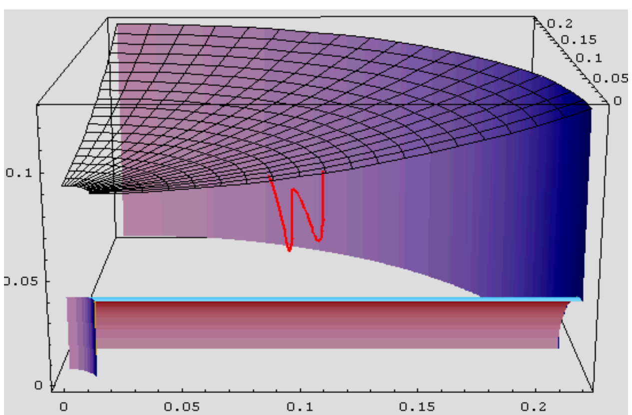
Multipacting

- Spoke cavity; full 3D structure
  coupler is connected to the cavity main body
  → should be checked very carefully
  (will not be a showstopper)

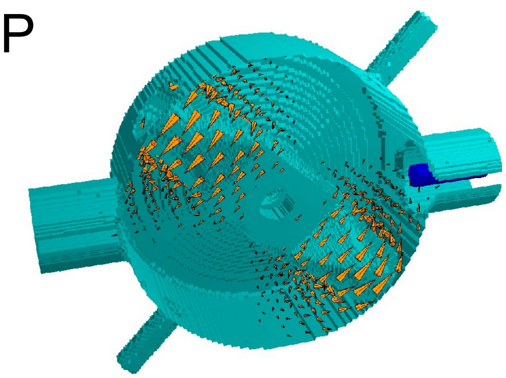
- 3D MP codes are under development and/or available now.

- Benchmarking or comparison with experiments are strongly needed

RIA prototype 3D MP simulation with MultP



(Courtesy of L. Kravchuk (INR))



AAA/LANL β=0.175, 350 MHz
Spoke cavity with
Power coupler (coupling concept)
Courtesy of F. Krawczyk (LANL)

TM Structures – Positive Features

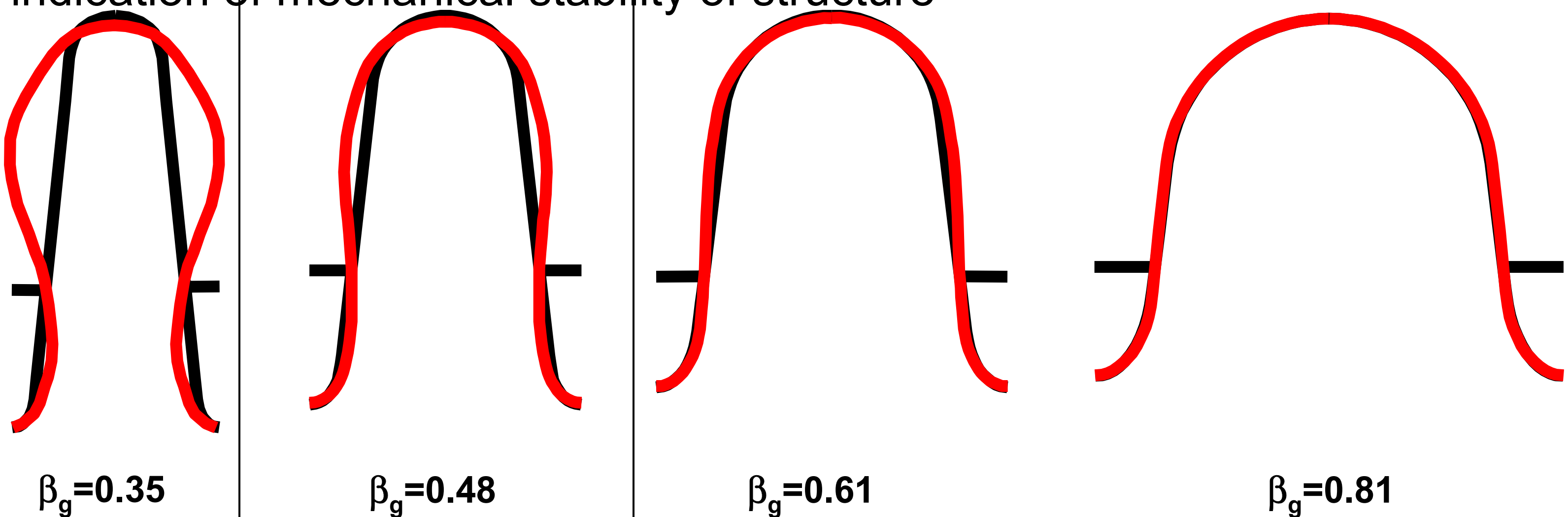
- Geometrically simple
- Familiar
- Large knowledge base
- Good modeling tools
- Low surface fields at high β
- Small number of degrees of freedom

$\lambda/2$ Structures – Positive Features

- **Compact, small size**
- **High shunt impedance**
- **Robust, stable field profile (high cell-to-cell coupling)**
- **Mechanically stable, rigid (low Lorentz coefficient, microphonics)**
- **Small energy content**
- **Low surface fields at low β**
- **Large number of degrees of freedom**

How Low Can We Go with β_g in TM Cavities ?

- Static Lorentz force detuning (LFD) at EoT(β_g)=10 MV/m, 805 MHz (Magnification; 50,000)
- In CW application LFD is not an issue, but static LFD coeff. provides some indication of mechanical stability of structure



RF efficiency; x
Mechanical Stability; x
Multipacting; Strong possibility

Will work in CW
Pessimistic in Pulsed application
Would be a competing Region with spoke cavity

Suitable for all CW & pulsed applications
Recent test results of SNS prototype cryomodule, β_g=0.61; quite positive; piezo compensation will work

How High Can We Go with β_g in Spoke Cavities?

- What are their high-order modes properties?
  - Spectrum
  - Impedances
  - Beam stability issues
- Is there a place for spoke cavities in high-β high-current applications?
  - FELs, ERLs
  - Higher order modes extraction

Parting Words

In the last 30+ years, the development of low and medium β superconducting cavities has been one of the richest and most imaginative area of srf

- The field has been in perpetual evolution and progress
- New geometries are constantly being developed
- The final word has not been said

The parameter, tradeoff, and option space available to the designer is large

- The design process is not, and probably will never be, reduced to a few simple rules or recipes
- There will always be ample opportunities for imagination, originality, and common sense